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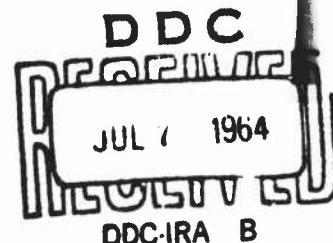
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THE DEVELOPMENT OF A WATER-SURFACE-BURST FALLOUT MODEL:  
THE RISE AND EXPANSION OF THE ATOMIC CLOUD

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by  
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## ABSTRACT

A water-surface-burst fallout model is being developed in three parts: (1) cloud rise and expansion, (2) particle formation, and (3) particle fall through the atmosphere. Part (1) is discussed in detail. The cloud is described by a set of equations for rate of rise, temperature, water content and other parameters, using an "entraining parcel" method. The equations allow for variations in atmospheric temperature, pressure and humidity as well as in initial cloud height, temperature, and air/water energy partition. Temperature and volume are calculated by extensions of standard meteorological equations. The key assumptions in the cloud model are:

1. The effective rate of flow of environmental air into the cloud per unit surface area is linearly proportional to the product of a characteristic velocity, and the ratio of cloud density to ambient density.
2. Cloud rise is retarded by both entrainment and an apparent eddy-viscous force, which is directly proportional to the same characteristic velocity and inversely proportional to the density ratio.
3. The lost kinetic energy of rise due to eddy viscosity remains in the cloud as turbulent kinetic energy, diluted by entrained air.
4. The characteristic velocity involved may be either the velocity corresponding to the sum of the kinetic energies of cloud rise and of turbulence, or the greater of absolute rate of cloud rise and average velocity of turbulence. Thus entrainment does not necessarily end when cloud rise ends, but continues as a turbulent diffusion.
5. The time required for the cloud to accelerate to its "terminal" velocity from rest is accounted for by an initial virtual mass term.

The equations have been programmed for machine computation. Comparison of numerical results with atomic-cloud observations indicates that a single empirical parameter, the entrainment constant  $\lambda$ , (which is the proportionality constant in assumption 1) can also be used as a measure of the rate of generation of turbulence. Correct predictions are obtained for rate of rise, cloud size, final cloud height and the late horizontal expansion of high yield clouds. This last has not previously been predicted by a cloud model. Numerical results also indicate that for low yields, the effect of initial water content of the cloud is minor compared with that of atmospheric humidity while for high yields the opposite is true.

Finally, the particle-formation and particle-fall portions of the model development are summarized.

## SUMMARY PAGE

### The Problem

Land-surface and sea-water-surface nuclear explosions produce different kinds of fallout. Water-surface-burst fallout particles consist mainly of sea salt and water. There is no fallout model for water-surface bursts. Until now, fallout predictions for water-surface-bursts have essentially used land-surface-burst fallout models.

### Findings

NRDL is developing a water-surface-burst fallout model, in three parts. This report gives a detailed discussion of Part 1: Cloud Rise, and an Outline of Work; Part 2: Particle Formation, and Part 3: Particle Fall Through the Atmosphere. The cloud rise is described by a set of equations which specify: cloud height, size, temperature, water content, etc., as functions of time and of initial and atmospheric conditions. The equations have been programmed for machine computation and the numerical results are in general agreement with weapon-test observations of atomic clouds.

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## SECTION 1

### INTRODUCTION

#### 1.1 PROBLEM AND OBJECTIVE

Land-surface and sea-water-surface nuclear explosions\* produce different kinds of fallout. Sea-water-surface burst fallout particles consist mainly of sea salt and water. Their water contents, and therefore, their masses, change during their fall. Unlike some land-surface-burst fallout particles (namely unvaporized soil particles) they do not exist as particles before the burst.

There is no fallout model for water-surface bursts. Until now, fallout predictions for water-surface bursts have been made using land-surface models, with at most minor modifications. (See Sec. 1.3)

Therefore, NRDL is developing a water-surface-burst fallout model. This report gives a detailed discussion of the rise and expansion of the atomic\*\* cloud from a water-surface burst, and an outline of work on the other parts of the problem.

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\* A water-surface burst is a nuclear explosion centered so close to the water surface that the fireball intersects the air-water interface, and the nuclear cloud initially contains both air and vaporized water.

\*\*The terms "atomic cloud" and "nuclear cloud" are used interchangeably in this report.

## 1.2 BACKGROUND AND APPROACH

Experimental data on water-surface-burst fallout is limited to cloud-rise histories and particle collections from two test shots at the Pacific Proving Grounds. Both shots were fired on barges in relatively shallow water, and the characteristics of the fallout may have been significantly influenced by the presence of barge and bottom material. The particle-collection data are scattered, and some of them are questionable. These data are insufficient for constructing an empirical fallout model. Therefore, the present model is being developed from fundamental physical considerations using the available experimental data as guideposts.

The development has been divided into three natural inter-related parts:

1. Cloud rise and expansion. (The main subject of this report.)
2. Fallout-particle formation in, and dispersion from, the cloud.
3. Fallout-particle fall through the atmosphere, subject to meteorological conditions.

Preliminary reports on the three parts are being published separately. The three parts will then, allowing for revision, be combined in the fallout model and computer program. Refinement and changes will be considered for incorporation into the model, as they become available.

Since the principal objective of a fallout model is to describe the arrival of radioactivity in the lower atmosphere and at the surface, Parts 1 and 2 of the development exist primarily to provide a scientific basis for this description.

This is a research fallout model. Numerical computations will show which variables have only minor effect on the results. For operational use, these results can then be approximately expressed in simplified equations, or in tables, graphs or nomograms.

Another summary of previous cloud models, including those based on the work of Machta<sup>1</sup>, Sutton<sup>2,3</sup>, and Taylor<sup>4</sup>, (see also the discussions in Refs. 5, 6, 7) will not be given. Instead, an attempt will be made to satisfy the present requirements for a description of cloud rise (Sec 2.1).

A completely analytic approach to cloud rise seems likely to bog down in hydrodynamics and to result in delay in delivering useful results. Therefore, this report uses a mixture of theoretical analysis and empiricism.

### 1.3 POSSIBLE INTERIM WATER-SURFACE-BURST FALLOUT PREDICTION METHODS

The following interim methods have been used or considered for prediction of water-surface-burst (WSB) fallout:

1. Use a land-surface-burst (LSB) model such as the NRDL D-Model<sup>8</sup> and accept the inaccuracies involved.
2. Make small adjustments on a LSB model - e.g., assume a total water-surface-burst activity equal to 80% of that from an LSB.<sup>9</sup>
3. Use the D-Model but with a particle size distribution that simulates WSB particle arrival times and locations (at sea level), although particle mass is fixed. Table 1.1 gives typical log-normal distributions of radioactivity vs particle size, used with the D-Model for Nevada soil and coral soil, and for salt particles.

Table 1.1

#### Log-Normal Distributions of Radioactivity vs Particle Size

Soil	Mean Particle Diameter, $\mu$ (Microns)	Log $\mu$	Standard Deviation of Log $\mu$	Particle Density (gm/cm <sup>3</sup> )
Nevada	113	2.053	0.732	2.6
Coral	162	2.209	0.424	2.6
Salt I	122	2.086	0.177	2.08
Salt II	85.5	1.932	0.168	2.08

In computing particle falling rates using the "Salt" distribution in the D-Model, the particles are taken as spheres, while Nevada and Coral particles are of irregular shape (cylinders with a 2:1 ratio of length to width).

4. Use the D-Model rate of rise and expansion but consider variation of water mass (and therefore, of falling rate at a given altitude) of slurry particles, using the computer program<sup>10</sup> developed for Farlow's particle-fall model.<sup>11</sup> If fallout particles consist of water and salt, we must not only specify the salt-mass vs activity distribution but also initial water content. (The simplest choice is that all particles are initially at sea-water concentration). This question is considered in Sec. 4.

5. Use the D-model, but replace its rate of rise with those calculated for a water-surface burst, using the equations of the present report.

6. Make the changes in the D-model described in both 4. and 5. above.

## SECTION 2

### THE RISE OF THE ATOMIC CLOUD: DISCUSSION OF PROBLEMS

#### 2.1 REQUIREMENTS FOR A DESCRIPTION OF CLOUD RISE

The input to be specified consists of:

1. Explosion energy and energy remaining in the fireball.
2. Height of burst (if bursts other than those exactly at sea level are considered).
3. Atmospheric temperature, pressure, and humidity as functions of altitude.

The desired output consists of the following cloud properties as functions of time:

1. Height.
2. Rate of rise.
3. Size (vertical and horizontal diameters).
4. Shape (spherical, ellipsoidal, etc.).
5. Mass.
6. Volume.
7. Water content (vapor and liquid or solid).
8. Temperature (either mean or with a gradient structure).
9. Turbulence characteristics.

The cloud-history output is needed as input to Part 2 of the fallout model: particle formation. This may then have to feed back to Part 1, for instance if the formed salt particles have removed a large part of water content from the cloud before its maximum height is reached.

The effect of wind on cloud rise is not considered in this report. It may be considered in a later report as outlined in Sec. 4.1.

## 2.2 PARCEL METHODS, LOCAL METHODS, AND SIMILARITY RULES

### 2.2.1 Parcel Methods and Local Methods

Parcel methods of describing the rising cloud treat the cloud as a whole, as if all parts of the cloud had the same properties (temperature, density, etc.). Any compensating down flow of air outside the cloud is neglected.

Local methods give equations for velocity and temperature at each point in the flow fields. They are mathematically complicated even when the field has axial symmetry, and typically describe the cloud by a set of partial differential equations. (Such equations, of course, refer to mean quantities, not turbulent fluctuations at each point.) Not only is it simpler to use an "entraining parcel" to represent the cloud, but also the extreme turbulence prevailing in the cloud makes it reasonable that the effect of entrainment is immediately averaged over the cloud. Although relative velocities of parts of the cloud are unspecified, turbulence parameters can still be used to represent the net effect of velocity fluctuations.

To sum up, local methods require numerical solution of partial differential equations, unless some similarity rules (see Sec. 2.2.2) can be devised. Any such similarity rules would restrict the ability to test the effect of variations in the atmosphere on the air/water composition of the cloud, on specific heat, etc. Such testing is one of the objectives of this "research fallout model." Parcel methods require only ordinary differential equations. Therefore, the following discussion (remainder of Sec. 2) and equations (Sec. 3) use a parcel method. Use of a parcel method does not keep us from discussing entrainment rules, viscosity analogies, etc.

### 2.2.2 Similarity Rules

If two sets of functions, with respect to both position and time, of velocity, density, temperature, etc., can be made identical by an appropriate choice of units of length, time, mass, and temperature,

they are said to be "similar"<sup>12</sup> This similarity is lacking in the rise of clouds from explosions of different yields, even though our parcel method replaces the continuous flow field by the use of average cloud and ambient values.

1. The atmospheric temperature-vs-altitude gradient changes at the tropopause so that low- and high-yield clouds encounter different conditions.

2. Even if the atmosphere has a constant temperature gradient (as in a simple model of the troposphere) the large temperature and density differences between cloud and ambient air violate the usual approximate similarity conditions.<sup>12</sup> Also, variation of specific heat with temperature may be important.

Therefore no attempt is made in this report to scale cloud rise from one set of initial and atmospheric conditions to another (although it is assumed that the "characteristic length" involved in the entrainment and momentum-change processes discussed in Sec. 2.6 is proportional to cloud radius). Instead the cloud rise equations (Sec. 3) are solved numerically for each set of parameters. Approximate similarities may then be noted in the solutions.

#### 2.2.3 Potential and Virtual Quantities

The use of potential (adiabatic equivalent) temperature is only possible if constant specific heat is assumed. (Potential temperature is the temperature air at a given height and pressure would assume if it were expanded or compressed adiabatically to some standard pressure, say sea-level pressure. If the effect of any water vapor or liquid or solid water in the air is neglected, the corresponding change of temperature with height is called a dry-adiabatic change.) Also, air entrainment and the presence of water vapor complicate the use of potential temperature. (If the presence of water in the air is taken into account, the adiabatic change of temperature with height, for saturated air, is called wet-adiabatic or saturated adiabatic. Because of the

release of latent heat of condensation, the wet-adiabatic rate of decrease of temperature, or lapse rate, is less than the dry-adiabatic lapse rate. See for instance Ref. 13.)

It is convenient to express some cloud properties in terms of virtual temperature (the temperature of dry air having the density of the actual humid air at the given temperature), when the water-vapor mass fraction of the air under consideration is small (so that, for instance, the specific heat of the air can be taken as that of dry air). But that this mass fraction is small is precisely what we cannot assume for a water-surface-burst cloud. We still need the actual temperature of the cloud, although we also use the expression for the virtual temperature, as a convenient algebraic abbreviation.

### 2.3 INITIAL CLOUD CONDITIONS

#### 2.3.1 Thermodynamic Conditions

In this study, consideration of the fireball begins when it is in approximate pressure equilibrium with the atmosphere.<sup>14,15</sup> (After the "fireball" starts to rise, we shall call it the "cloud.") The temperature at this time (initial temperature) is 3000°K to 4000°K. By not going back to higher temperatures, we avoid having to consider air dissociation and loss of energy by thermal radiation. (The cooling of the fireball from 4000°K to 3000°K occurs in a few seconds even for a megaton shot.)

For a burst centered near the water surface, part of the fireball gas is vaporized sea water and part is atmospheric air. Immediate mixing of the air and salt-steam volumes is assumed. This mixing may be aided by the greater buoyancy (lower molecular weight) of water vapor rising into the hot air. Given the fireball energy and temperature, the air-water energy partition, and the specific heats of air and water as functions of temperature, the initial cloud radius and mass can be calculated by conservation of energy, or rather enthalpy. (Appendix B.3, Initial Conditions.) It is assumed that the dry air and water vapor

in the cloud satisfy the perfect-gas law, but that their specific heats may vary with temperature.

In using perfect-gas theory, we are not claiming the fireball was heated reversibly; of course it was not. We are estimating the amount of stored energy. To use an analogy, we are interested not in the fuel spilled in loading a rocket, but in how much is on board.

We assume the initial cloud form is a sphere. For a water-surface burst the initial conditions could center the "spherical" fireball at water level, or, alternately, at a height equal to the initial cloud radius so that the fireball just touches the surface, or, more reasonably, somewhere between. It is only for megaton yields, for which the initial cloud radius is a kilometer or more, that this variation in initial height of cloud center is large enough to have a significant effect on final cloud height and size.

### 2.3.2 Air-Water Energy Partition

The calculated initial cloud size is not very sensitive to the assumed air-water energy partition. An all-air cloud has about 10% greater radius than one with a 50-50 energy partition, because the greater specific heat of water and the large latent heat of evaporation overcompensate for the smaller molecular weight of water as compared to that of air, and result in a smaller fireball.

One might expect that a check on the air-water partition could be made by observing the initial size and rate of rise of the cloud from a water-surface burst as compared with a land-surface burst or a low air burst of the same yield. But calculations with the equations in Sec. 3 indicate that for a megaton surface burst, the differences in cloud height and radius of a 50-50 burst as compared with those of a 100%-air burst near the surface, are too small to be observed on photographs, if the differences are due solely to the energy partition. Therefore, any observed differences must be due to other factors. (Different initial temperatures are one possibility.)

## 2.4 EFFECT OF WATER CONTENT ON THE CLOUD

### 2.4.1 Effect of Water Content on Cloud Rise

The importance of the sea-water content of the fireball is due to:

1. The accompanying sea salt which condenses and coagulates to form fallout particles (second part of this problem).

2. The possible additional thrust the cloud receives relatively late in its rise from release of latent heat of condensation, and of freezing. Once the rising cloud has cooled to below condensation temperature, the temperature - height curve (neglecting entrainment) is wet-, not dry-, adiabatic (Sec. 2.2.3), so that the cloud then cools, and loses buoyancy, more slowly. Energy release by condensation appears to be significant only for low-yield clouds (most of whose water comes from the atmosphere). Calculations indicate that megaton clouds reach the stratosphere before cooling to the boiling point, so the extra thrust may come surprisingly late. But the water fraction of the cloud is then so small as to make the thrust insignificant, unless the cloud has a hot core or ring, which is not considered here.

The influence of the salt content of sea water (3% by mass) on cloud density, temperature, and water-vapor pressure is neglected.

Atmospheric humidity also influences cloud rise. Computations with the cloud-rise equations given later in this report indicate for a 1 KT cloud, entrained atmospheric water vapor is far more important than initial water content, while for a 1000 KT (1 MT) cloud, the opposite is true. The reason for this difference is that the cloud surface-to-volume ratio decreases as yield and cloud size increase, so that entrainment, which is proportional to this ratio (Sec. 2.6.1), becomes less influential.

Computations indicate that water-vapor condensation begins below the freezing point ( $273^{\circ}\text{K}$ ) for megaton clouds, so that the latent heat released may include that from the liquid-to-ice transition. A choice as to which latent heat to use is complicated by the presence of salt

and by turbulence, both of which influence the freezing point. One must ask the question: does the 15% additional latent heat of freezing influence cloud rise much? This will be tested by computation.

Possibly the radar image of a water surface burst cloud would look "denser" than that of a non-water burst. This is an interesting subject for future investigation.

#### 2.4.2 Effect of Water Loss on Cloud Buoyancy

In the later stages of cloud rise, much of the water content is in liquid or solid form. The question arises: does all liquid and solid water remain in the cloud, or is some of this "spent fuel" jettisoned, lightening the cloud's load?

If the water-and-salt particles coagulate to a size with a sensible falling rate while the cloud is still rising (this will be discussed in detail in a later report), then there is a gain in cloud buoyancy by loss of these particles. The maximum possible influence of this effect is measured by the ratio of liquid and solid water mass to dry air mass in the cloud. A small value of the water percentage of the fireball energy partition, together with a large entrainment rate and a low-humidity atmosphere, would minimize the possible gain in buoyancy. If this fallout from the cloud takes place after the end of cloud rise, it should have little effect on cloud buoyancy because:

1. The cloud has gained so much air by entrainment that the water contribution to total density is small.
2. The cloud no longer moves as a unit. See Ref. 16, (Sec. 13, Settling of Clouds) for a criterion for this mass-subsidence problem.

#### 2.4.3 Effect of Water Loss on the Cloud's "Heat Engine"

Can an MT cloud, rising past its equilibrium level in the stratosphere against a positive ambient temperature gradient, have lost so much of the condensed (and coagulated) water by fallout, or have entrained so much dry air, that it is then cooling dry-adiabatically? Calculations indicate this can occur, but since dry and wet adiabatics hardly differ, at these low temperatures, the effect on cloud motion is negligible.

## 2.5 INFLUENCE OF VARIATION OF AMBIENT CONDITIONS

The volume and mass of a rising cloud change by two processes:

1. Adiabatic expansion.
2. Entrainment of ambient air.

How sensitive to variations in meteorological conditions are these processes? The first process depends on atmospheric pressure, which does not vary much (at a given altitude). For high-yield clouds, most of the second process would occur at greater altitudes where time variations of meteorological conditions are smaller than in the lower troposphere. But as mentioned in Sec. 2.4.1 low-yield clouds, on the other hand, are sensitive to ambient humidity.

While theories of atomic cloud behavior have largely been suggested by studies of cumulus cloud behavior, there are limitations on the use of such meteorological theories to describe atomic clouds. Cumulus clouds have been assumed to consist of a series of "thermals" (buoyant convective elements). The atomic cloud is a single thermal. Some orders of magnitude are quite different for atomic clouds and cumulus clouds. The initial temperature difference between a cumulus cloud and its environment is about  $1^{\circ}\text{C}$  or less. This is enough to move the cloud up, and even bounce it off the tropopause and back down several times, according to some theoretical calculations.<sup>17</sup> Nuclear clouds have tremendous initial temperature differences, much greater speeds of rise and, for megaton bursts, much greater sizes than cumulus clouds.

Thus, atmospheric temperature conditions are much less important in the early stages of cloud rise for nuclear clouds than for cumulus clouds. If the cloud temperature is  $1000^{\circ}\text{K}$ , it does not immediately matter whether ambient temperature is  $290^{\circ}\text{K}$  or  $300^{\circ}\text{K}$ . The assumed initial rate of rise of the cloud makes a big difference in the subsequent rise of a "thermal" on the meteorological scale<sup>17</sup> but has negligible influence on a nuclear cloud. Hence, we can assume that a nuclear cloud (or fireball) is still at rest when it has cooled to our initial temperature (about  $3000^{\circ}\text{K}$ ). Also, unlike the case of cumulus cloud rise, ambient turbulence can be neglected, and surely so can "erosion" (evaporation at the surface of the cloud) except for very low yields or after stabilization.

## 2.6 A MODEL OF ENTRAINMENT AND MOMENTUM CHANGE

Several approaches to cloud rise were considered in the course of this study. These approaches included:

Static methods:

Thermal equilibrium

Potential-kinetic energy balance

Dynamic (momentum-equation) methods:

Potential flow

Drag coefficient

External or internal momentum exchange.

The various approaches are discussed in Appendix B. The internal-momentum-loss rule and the closely related entrainment rule that were adopted are discussed in this section.

### 2.6.1 Entrainment

It was suggested by Taylor<sup>4</sup> that the rate of increase of cloud mass due to entrainment of air is proportional to surface area times absolute rate of cloud rise:

$$\frac{dm}{dt} = S \lambda |u| \rho_e \quad (2.1)$$

where m is cloud mass

t is time

S is surface area

$\lambda$  is a dimensionless constant of proportionality

$u = \frac{dz}{dt}$  is rate of rise, where

z is cloud (center) height

$\rho_e$  is ambient (environment) density

This equation is obtained by assuming that, averaging over the cloud surface, S, the ambient air, of density  $\rho_e$ , flows "onto" (or into) the cloud at a rate proportional (factor  $\lambda$ ) to the absolute value of rate of rise,  $|u|$ . (The absolute value  $|u|$  used here and later avoids negative entrainment or drag, if the cloud sinks down after overshooting

its equilibrium height.) For a spherical cloud of radius  $r$ , this leads to  $dr = \lambda |dz|$  if the change in cloud density is neglected. Empirical values of  $\lambda$  for nuclear and other clouds range from 0.07 (Ref. 7) to 0.30 (Ref. 5) and 0.34 (Ref. 18). Theoretical and experimental work on entrainment has considered mostly cumulus clouds and turbulent jets, at about the same density and temperature as ambient air. In actuality, the rising cloud is initially about ten times as hot and one tenth as dense, as the ambient air. We propose, therefore, that the entrainment rate is also proportional to the "relative inertia" of cloud and ambient air, i.e., the density ratio  $\rho/\rho_e$ , (where  $\rho$  is the cloud density) so that,

$$\frac{dm}{dt} = S \lambda \frac{\rho}{\rho_e} |v| \rho_e = \frac{S}{V} m \lambda |v| \quad (2.2)$$

where

$V$  = cloud volume

$v$  is a characteristic velocity given by:

$$v = \sqrt{u^2 + 2E_k}$$

where  $E_k$  is the turbulent energy density (Sec. 2.6.2). This entrainment rate agrees qualitatively with the remark in Ref. 18 that "in the early stages of its motion the atomic bomb cloud behaves more as a bubble of air in water in which case there is no entrainment at all." Also, weighting the entrainment rate by the density ratio avoids the difficulty encountered by the authors of Ref. 7. So as not to cool the cloud too fast they had to make  $\lambda$  "surprisingly small." Then the choice of initial radius required to make the final cloud size correct was found to be "surprisingly large" and was described as an "artificial radius."

Any justification for introducing the density ratio is necessarily heuristic, but an argument for this approach can be made in mechanical terms. Suppose that a lump of cloud gas, with velocity  $u$ , collides with

a lump of stationary ambient air and imparts a velocity to it, and suppose that the corresponding momentum and kinetic energy are conserved in a collision. Momentum is not conserved in the overall cloud-atmosphere interaction, since turbulence is generated at the expense of directed motion, but we can imagine the interaction as consisting of separate momentum-transfer and momentum-dissipation processes. Suppose now that a lump of ambient air must gain a certain threshold velocity  $ku$  to be entrained, (where  $k$  is a constant), and that the lump of cloud gas remains in the cloud. We calculate the volume of ambient air  $v_e$  to which a unit volume of cloud can transfer this threshold velocity. (Actually the cloud lump may make many collisions, but we idealize the process to a single collision.) The equations for conservation of momentum and kinetic energy are, respectively,

$$\rho u = \rho u' + \rho v_e v_e ku \quad (2.3)$$

$$\frac{1}{2} \rho u^2 = \frac{1}{2} \rho u'^2 + \frac{1}{2} \rho v_e v_e k^2 u^2 \quad (2.4)$$

where  $u'$  is the velocity of the cloud lump after collision.

$$\text{The solution is } v_e = (\rho/\rho_e)(2/k - 1) \quad (2.5)$$

$$u' = u (k - 1) \quad (2.6)$$

(This solution has no physical meaning for  $k > 2$ .)

Thus, the volume (and consequently the mass) of ambient air, that a lump of cloud of unit volume can entrain with a given threshold velocity, is proportional to the density ratio  $\rho/\rho_e$ . We could also say that the distance that the lump travels before losing its momentum is proportional to  $\rho/\rho_e$ , and call this distance the "characteristic length." This length could then be considered the distance between the inside and the outside of the cloud.

In the particular case,  $k = 1$ , the threshold velocity is  $u$  itself, the entraining cloud lump is left with zero velocity, and the entrained volume is  $\rho/\rho_e$  times the entraining volume, so that the entrained mass equals the entraining mass. Now suppose that in time  $dt$  a volume of cloud gas  $S \lambda |v| dt$  "flows out" of the cloud. (This is the Taylor hypothesis<sup>4</sup> applied to the entraining air instead of the entrained air.) Then the entrained mass is

$$dm = \rho S \lambda |v| dt, \quad (2.7)$$

so that

$$\frac{dm}{dt} = \frac{S}{V} m \lambda |v| dt \quad (2.2)$$

as was suggested above.

More formally, we can estimate the entrainment rate by regarding momentum as a diffusible quantity. In a turbulent flow, however, ordinary molecular diffusivity is of minor importance compared with the diffusive effect of turbulence. As has been done by many investigators, we suppose the diffusive effect of turbulence can be expressed as an "eddy diffusivity" having the same dimensions as molecular diffusivity, although the eddy diffusivity is a property of the flow conditions, not of the fluid. Just as molecular diffusivity is the product of the speed and mean free path of a molecule, times a constant, we may take the eddy diffusivity to be proportional by a factor  $\lambda$ , to the product of the speed,  $v$ , of a lump of cloud, and a length,  $\ell$ . We call  $v$  and  $\ell$  the "characteristic speed" and "characteristic length," respectively.

Suppose the characteristic speed,  $v$  to be the speed corresponding to the total kinetic energy of a lump of cloud. This energy consists of the kinetic energy of rise and the kinetic energy of turbulence,  $E_k$ , so that

$$v = \sqrt{u^2 + E_k} \quad (2.8)$$

where the positive value of the square root is taken.

Suppose the characteristic length,  $\ell$ , to be proportional to cloud (vertical) radius, and that this length is the distance from inside to outside the cloud. Then, assuming that the rate of change of momentum per unit volume with distance along the length  $\ell$  has the average value  $\rho v / \ell$ , the total outward flow of momentum in time  $dt$  is  $S(\lambda v \ell) (\rho v / \ell) dt$ , (where  $\lambda v \ell$  is the eddy diffusivity), so that (dividing the momentum flow in  $dt$  by the momentum per unit volume  $\rho v$ ) the corresponding change in volume is  $S\lambda v dt$ , as assumed above in Eq. (2.2). The boundary of the cloud moves outward, as a result of this increase in volume. As outlined in Sec. 2.6.3, entrainment continues after the end of cloud rise, when there is no longer any net total cloud momentum, and now  $v$  consists entirely of the average velocity of turbulence,  $\sqrt{2E_k}$

The following analogy to the entrainment process was proposed by C. F. Ksanda. Consider the rising cloud, of radius  $r$ , as a piston of the same radius. Let the cloud rise during an interval  $dt$ . The resulting environment process consists of two steps.

- (1) Neglecting expansion, an element of cross sectional area  $dA$  at the top of the cloud rises a distance  $dz$ , and sweeps out a volume  $dAdz_1$ .
- (2) Suppose that the ambient air in this volume is engulfed by the cloud. Its mass is  $\rho_e dAdz_1$ . If the entrained mass is small compared to the cloud mass  $m$ , it now expands from  $dAdz_1$  to  $dAdz_1 (\rho_e / \rho)$ , using the gas law at constant pressure. The total increase in cloud height in time  $dt$  is thus not  $dz_1$  but  $dz = (\rho_e / \rho)dz_1$ .

Representing the top surface of the cloud by its horizontal projection A, the total entrained mass is  $dm = \rho_e A dz_1$ , and we can write

$$\frac{dm}{dt} = \rho_e A \frac{dz_1}{dz} = \rho_e A \frac{dz_1}{dz} \frac{dz}{dt} = \rho A u \quad (2.9)$$

and since  $m = \rho V$

$$\frac{dm}{dt} = m \frac{A}{V} u \quad (2.10)$$

For a sphere,  $A = S/4$  where  $S$  is the surface area. This analogy thus predicts  $\lambda = .25$ , which is actually in good agreement with observation. The analogy does not of course apply in the late stages of entrainment when the driving force is not cloud rise but internal turbulence, but at that time cloud and ambient densities are nearly equal.

### 2.6.2 Momentum Loss and Turbulent Kinetic Energy Gain

To determine momentum loss, we draw an analogy between turbulence and molecular motion, as we did for entrainment. In this case, however, we speak of eddy viscosity instead of eddy diffusivity.

Suppose that the rising cloud is retarded by the stationary ambient air, as if the latter, like the cloud, were an "eddy-viscous" fluid, with the same eddy viscosity  $\lambda v \ell$ . Then the force on a unit area of cloud surface is proportional to  $\rho_e \lambda v \ell (u/\ell)$ , where the actual velocity gradient is replaced by an average value  $u/\ell$ . The total force is then proportional to  $S \rho_e \lambda v \ell (u/\ell)$  and the force per unit cloud mass to  $(S/V)(\rho_e/\rho) \lambda v u$ .

It may be argued that not the total surface area but the projected area normal to the direction of rise should be used. For a sphere, the ratios of these quantities to  $V$  are respectively  $3/r$  and  $3/4r$ . Furthermore, we do not know whether eddy diffusivity and eddy viscosity are the same, or merely proportional. Therefore, for

generality, we replace  $S/V$  by a constant times  $1/r$ , combine the constant with  $\lambda$  and write the product as  $2k_2$ , so that

$$\text{force per unit cloud mass} = 2k_2 \frac{\rho_e}{\rho} \frac{\lambda v u}{r} \quad (2.11)$$

Computations indicate that when this form is used the empirically correct value of  $2k_2$  is very nearly the same  $\ell$  appearing in the entrainment equation.

If the retarding force on the cloud were an ordinary viscous force, the kinetic energy loss due to this force would appear as increased heat, i.e. random motion of molecules. Since it is an eddy-viscous force, the lost energy appears as an increase in turbulent kinetic energy, i.e. random motion of lumps of fluid, which are supposedly mixed into the cloud. Thus, eddy viscosity does not reduce the total cloud kinetic energy, but converts some of the energy of rise to turbulent energy.

Then, neglecting entrainment, the rate of increase of turbulent kinetic energy per unit mass is equal to the rate of decrease of kinetic energy of rise due to eddy viscosity,

$$\frac{dE_k}{dt} = - k_2 \frac{\rho_e}{\rho} \frac{\lambda v u^2}{r} \quad (2.12)$$

We regarded the eddy viscous momentum loss as occurring on the outside of the boundary of the cloud, in fluid of density  $\rho_e$ . The same final result could be obtained by regarding the momentum loss as occurring on the inside of the boundary, in fluid of density  $\rho$ , and with a different velocity gradient given by  $u/(\ell \rho/\rho_e)$ . This would agree with the comment in Sec. 2.6.1 that the distance that a lump of cloud travels while transferring momentum is proportional to  $\rho/\rho_e$ . This suggests that if we follow the original Taylor entrainment rule, Eq. (2.1) in Sec. 2.6.1, we should then omit the factor  $\rho/\rho_e$  in the eddy-viscous force.

That is, the present model gives more drag and less entrainment by the factors  $\rho_e/\rho$  and  $\rho/\rho_e$  respectively, than the "original" rule. These arguments are, however, suggestive rather than convincing, and equally plausible ones might be made to obtain other results.

Qualitatively, the net effect on cloud rise of introducing the density-ratio hypothesis is:

1. The cloud cools more slowly since it entrains relatively less low-altitude air and more high-altitude air whose potential temperature (Sec. 2.2.3) is higher.
2. The early rate of mass increase is less.
3. There is relatively less drag by entrainment and relatively more by eddy viscosity, so that more turbulence is generated.

We can now equate the rate of change of cloud momentum, per unit mass, to the sum of the forces acting on the cloud.

$$\frac{du}{dt} = \frac{\rho_e - \rho}{\rho} g - 2k_2 \frac{\rho_e |v| u}{\rho r} - \frac{l}{m} \frac{dm}{dt} u \quad (2.13)$$

Here the three terms on the right are respectively due to buoyancy (i.e. Archimedean force), drag and entrainment. In the momentum equation in Sec. 3.1 the  $r$  in the denominator of the drag term is replaced by a generalized characteristic length  $\ell$ , which assumes different values for spherical and spheroidal clouds.

If the total drag force  $F_D$  on a spherical cloud is represented by a drag coefficient  $C_D$ , then

$$C_D = \frac{F_D}{\pi r^2 (\rho_e u^2 / 2)} \quad (2.14)$$

so that the drag force per unit cloud mass is given by

$$F_D/m = C_D \frac{3}{8\pi} \frac{\rho_e}{\rho} u^2 \quad (2.15)$$

which has exactly the form of the drag term in the momentum equation, Eq. (2.13).

Finally, we can allow for the time for the cloud to accelerate from rest to terminal velocity by treating the initial motion of the (spherical) cloud as a potential flow, which sets in motion a volume of ambient air equal to one-half the initial displaced volume. All entrained fluid becomes turbulent, so that the initial cloud is presently "dissolved in turbulence"<sup>4</sup>. Therefore we multiply the entire right side of the momentum equation by  $\frac{m}{m+m'}$ , where  $m'$  is a virtual mass equal to the mass of one half the initial displaced volume. This virtual mass factor is derived in App. B.2.1.

### 2.6.3 Additional Effects of Turbulence

When cloud rise ends, no more kinetic energy can be transferred to turbulence. The turbulence drives the apparent horizontal expansion of the cloud which continues (for high-yield clouds) after the end of the rise, vertically stabilized by the stratospheric temperature inversion (See Sec. 2.8). In our parcel method, using the characteristic velocity:

$$v = \sqrt{u^2 + 2E_k}$$

in the entrainment equation, Eq. (2.2), this late expansion corresponds to the case  $u = 0$ ,  $v = \sqrt{2E_k}$

If instead we chose as characteristic velocity,

$v = \max(|u|, \sqrt{2 E_k})$ , then the idealized picture would be that as cloud rise slows down, the diffusive spread of turbulence (turbulence-induced mixing) overtakes the radial entrainment flow induced by the mean flow.

During cloud rise and expansion, the turbulence influences coagulation of fallout particles in the cloud.<sup>19</sup> Thus, retardation of cloud rise, late horizontal expansion of cloud, and particle growth give us three check points for the constant  $k_2$ , (Sec. 2.6.2), which governs the rate of transfer of kinetic energy from directed motion to turbulence. As noted in Sec. 2.6.2, computation indicates that  $2k_2$  is related to the entrainment parameter  $\lambda$ , so that  $\lambda$  is the only independent empirical quantity in the description of the cloud. It is assumed that only a small fraction of the turbulent energy is degraded to heat by viscosity during cloud rise, i.e. relaxation time is large compared to cloud rise time. Computations of the late horizontal expansion of megaton clouds confirm this hypothesis.

#### 2.6.4 Compressibility Effects on Cloud Rise

For yields over 15 MT, the present equations (Sec. 3), as well as potential flow methods, predict cloud-rise speeds that may be too high, if the drag coefficient or eddy-viscosity coefficient used is the same as that used for lower yields. For a 100 MT cloud, speeds as high as sonic speed, at ambient temperature, are predicted.

We have assumed that the "characteristic length" increases linearly with cloud radius. But at some large radius and speed of rise, it would seem that a small part of the boundary no longer knows what the cloud diameter is, i.e. has time to adjust itself to an increase in diameter. That is, the time for information to pass half way around the periphery is not small compared with the time for the cloud to rise a significant distance, say one radius, i.e.  $\frac{\pi r}{c}$  is at least as large as of  $\frac{r}{u}$ , say  $\frac{u}{c} \geq \frac{1}{\pi}$ , where  $c$  is the speed of sound.

Compressible flow theory suggests multiplying the eddy-viscous drag by a factor  $\frac{1}{\sqrt{1 - M^2}}$ , where  $M$  is the Mach number,  $M = u/c$ .

(A more sophisticated correction would permit  $M = 1$ .) This suggestion is heuristic, and based on calculations for slender bodies at small

angles of attack. Furthermore, even  $M = 2/3$  only produces a 35% increase in drag, using this factor.

The local speed of sound near the boundary of the cloud is higher than in the ambient air, because of the high cloud temperature, so that the local Mach number is smaller, and the "correction" less effective. But also, the local flow speed near the cloud equator may be greater than the net upward speed, increasing the Mach number. These comments are offered only as notes for future investigation.

## 2.7 VORTICITY AND CIRCULATION

The details of vortex structure and cloud circulation are not considered in this study. We are not interested in vortex structure but in the total turbulent energy arising from eddy-viscous momentum loss during cloud rise. We put this energy in a "black box" and let it contribute to the characteristic velocity given by Eq. (2.8) and thus to the entrainment process. Just what vortices are in the black box does not matter. In fact, the box contains the turbulent spectrum.

We do not use a vortex model such as that of Levine.<sup>20</sup> In his model the bubble exchanges mass with the environment. The narrowness of the wake of atomic clouds indicates very little such exchange takes place. And even this narrow wake may just be environmental air lifted above the condensation level without mixing. For the nuclear cloud case there is no inactive portion of the cloud through which the bubble is rising, since the whole cloud is the bubble. Unlike a cumulus cloud, it is not true that "the turbulence is very likely confined to a thin layer the thickness of which is small compared with that of the bubble." Furthermore, Levine's model uses an exchange factor (See App. B.2.3). In the nuclear cloud, all the retarding ambient air is entrained, (except possibly the "initial virtual mass" in our model) so there is no loss of total kinetic energy by exchange, but only transfer from mean motion to turbulence.

## 2.8 SPHERICAL AND NON-SPHERICAL CLOUDS

We assume that the initial cloud form is a sphere. If and when the top of an atomic cloud reaches or passes the tropopause it is observed to flatten (by horizontal expansion, not vertical contraction) into an oblate spheroid. We represent this flattening by postulating that vertical expansion ceases when the cloud center reaches the tropopause, so that the cloud becomes an oblate spheroid. The entrainment rule is unchanged. (Actually, even for clouds not passing the tropopause horizontal diameter grows somewhat more than vertical diameter.) What are the mechanics of this shape change?

1. Effects of the finite size of the cloud. The top and base of the cloud encounter different conditions. The larger the cloud, the less valid it is to approximate ambient conditions by those at altitude of the cloud center. Near the tropopause, the atmosphere has a non-linear lapse rate. When the cloud reaches the tropopause, its top loses buoyancy more rapidly than its bottom which is still below the tropopause. Picturing the cloud as an elastic body meeting increased resistance to its rise, we can say that to conserve momentum it can only move outward. This suggests that the rate of distortion is proportional to the difference in buoyancy between its top and bottom; or better, to the rate of change of this difference with altitude. That is, a sharp increase in relative stability in the atmosphere, or roughly,  $\frac{d^2T_e}{dz^2} > 0$ , as at the tropopause, flattens the cloud. (Here  $T_e$  is ambient temperature.) Speculatively,  $\frac{d^2T_e}{dz^2} < 0$  would produce a vertical elongation of the cloud into a prolate spheroid. This speculation is supported by a recent meteorological cloud rise study.<sup>21</sup> A computation was made for a conditionally unstable atmosphere, and "unlike the spherical shape envisaged in bubble theories, the cloud... develops into a tall and slender current." So we should say that

distortion rate is proportional to rate of change of stability. But for a nuclear cloud only a very sharp stability change, such as at the tropopause, can affect cloud form.

2. Turbulent diffusion. For a high-yield cloud, the accumulated turbulent kinetic energy induces turbulent (horizontal) diffusion of cloud particles with the vertical thickness fixed by the stably stratified atmosphere. The rate of horizontal "expansion" depends on the turbulent energy density and thus on the rate of eddy-viscous momentum loss during rise. (See Sec. 2.6.2.) Low yield clouds rise more slowly, generating less turbulence, and their energy density decreases rapidly by dilution by entrained air so that such clouds hardly expand after the end of rise.

## SECTION 3

### A SET OF EQUATIONS FOR CLOUD RISE

The following set of equations for cloud rise uses an "entraining parcel" method (Sec. 2.2) and the entrainment and eddy-viscosity concepts discussed in Sec. 2.6. The numbered equations are those forming the set, which have been solved numerically (Sec. 3.14). The unnumbered equations are used only in deriving the numbered equations. The alternate (A) equations correspond to the hypothesis that the "characteristic length," and therefore the rates of momentum change and entrainment, do not depend on the density ratio (Sec. 2.6). The set includes "dry" (D) and "wet" (W) equations for the water vapor content, temperature, volume, and liquid or solid water content of the unsaturated and saturated cloud, respectively. The momentum, water vapor and temperature equations are suggested by those used in parcel-method cumulus cloud calculations.<sup>17, 22</sup>

#### 3.1 MOMENTUM

As in Eq. (2.13), we equate the rate of change of momentum to the sum of buoyancy and eddy-viscous forces,

$$\frac{d}{dt}(mu) = v (\rho_e - \rho)g - \frac{2k_2 v u}{\ell_1} m$$

where  $t$  is time

$m$  is cloud mass

$u$  is rate of cloud rise

$V$  is cloud volume =  $m/\rho$

$\rho$  is cloud density

$\rho_e$  is ambient air density

$g$  is acceleration due to gravity

$k_2$  is constant

$v = \sqrt{u^2 + 2E_k}$ , where  $E_k$  is the turbulent energy density

$\ell_1$  is a "characteristic length" (Sec. 2.6)

Using the perfect gas law for air and water vapor since the pressure inside and outside the cloud is the same, then with appropriate weights,

$$\frac{\rho_e}{\rho} = \frac{Tq(x)}{T_e q(x_e)} \left( \frac{1+x}{1+x+w} \right)$$

where  $T$  and  $T_e$  are respectively cloud and ambient temperature

$q(x)$  may be shown to equal  $\frac{1+x/\epsilon}{1+x}$

$x$  and  $x_e$  are respectively, cloud and ambient "mixing ratios"

(ratios of water-vapor mass to dry air mass per unit volume)

$w$  is the ratio of liquid and solid water mass to dry air

mass in the cloud

$\epsilon = 18/29$ , the ratio of the molecular weight of water vapor to  
that of dry air

$\frac{1+x}{1+x+w} = \beta$  is the ratio of cloud gas density to total cloud  
density. (This term allows for the liquid and solid water  
content of the cloud, if any. The effect of this content  
is usually small.)

We use the symbols:  $T^* = Tq(x)$  and  $T_e^* = T_e q(x_e)$ .

$T^*$  and  $T_e^*$  are thus the virtual temperatures (Sec. 2.2.3) and  $T^* \beta$  is  
the (cloud) virtual temperature allowing for the contribution of liquid  
and solid water to the total cloud density.

Taking the characteristic length proportional to the density  
ratio so that  $\ell_1 = \ell \rho_e / \rho$ , where  $\ell$  depends only on cloud size, then

the momentum-balance equation may be rewritten as

$$\frac{du}{dt} = \left[ \frac{T^*}{T_{e^*}} \beta - 1 \right] g - \left[ \frac{2k_2}{\ell} \frac{v}{T_{e^*}} + \frac{1}{m} \frac{dm}{dt} \right] u \quad (3.1)$$

Alternatively, if  $\ell_1$  is independent of  $\rho/\rho_e$  so that  $\ell_1 = \ell$ ,

$$\frac{du}{dt} = \left[ \frac{T^*}{T_{e^*}} \beta - 1 \right] g - \left[ \frac{2k_2}{\ell} v + \frac{1}{m} \frac{dm}{dt} \right] u \quad (3.1A)$$

Since in potential-flow theory, energy and momentum-balance statements require that the fluid mass accelerated from rest include a virtual mass equal to one half the initial displaced mass, then the entire right side of each of Eqs. (3.1) and (3.1A) must be multiplied by  $\frac{m}{m + m_o T_o^*/2T_{eo^*}}$ , where the subscript o indicates the initial value of each quantity. (See App. B.2.1.)

### 3.2 HEIGHT

The height,  $z$ , of the center of the cloud is given by

$$\frac{dz}{dt} = u \quad (3.2)$$

### 3.3 WATER VAPOR

#### 3.3.1 Dry

During the "dry" (unsaturated) period no water is lost by condensation. In time  $dt$ , a mass  $dm$  of ambient air is entrained, so that, at time  $t + dt$ , the new "mixing ratio" will be given by

$$x(t + dt) = \frac{\frac{m}{1+x} + dm \frac{x_e}{1+x_e}}{\frac{m}{1+x} + \frac{dm}{1+x_e}}$$

from which, by the definition of the derivative,

$$\frac{dx}{dt} = - \frac{1+x}{1+x_e} (x - x_e) \frac{1}{m} \frac{dm}{dt} \quad (3.3D)$$

### 3.3.2 Wet

The cloud is now saturated, so that  $x_e$  does not influence  $x$ . Let  $p$  be the total pressure and  $p_w$  the pressure of water vapor in the cloud. Then

$$x = \frac{\epsilon p_w}{p - p_w}$$

From the hydrostatic equation, the gas law and the Clausius-Clapeyron equation:

$$\frac{dp_w}{dT} = \frac{L \rho_w}{T}$$

where  $\rho_w$  is the vapor density and  $L$  the latent heat of condensation, it follows that

$$\frac{1}{x} \frac{dx}{dt} = (1 + x/\epsilon) \frac{L\epsilon}{R_a T^2} \frac{dT}{dt} + (1 + x/\epsilon) \frac{g_*}{R_a T_e} u \quad (3.3W)$$

where  $R_a$  is the gas constant for dry air.

## 3.4 TEMPERATURE

### 3.4.1 Dry

The specific heat of entrained air is taken as that of dry air,  $c_{pa}(T)$ . For a small interval  $dt$ , the sum of the following two terms is zero (by conservation of energy):

Heat used to warm entrained air  $dm$ :

$$\left( \int_{T_e}^T c_{pa}(T) dT \right) dm$$

and

Heat absorbed in adiabatic expansion  $dp$ :

$$m \left[ c_p(T) dT - R_a T^* \frac{dp}{p} \right]$$

where  $c_p(T)$  is the weighted mean of the specific heats of air and water vapor,

$$c_p(T) = \frac{c_{pa}(T) + c_{pw}(T)}{1 + x}$$

The expansion term becomes, using the hydrostatic equation and the gas law,

$$m \left[ c_p(T) dT + \frac{T^*}{T_e^*} gdz \right]$$

Equating the sum of the two changes to zero,

$$\frac{dT}{dt} = - \frac{g}{c_p(T)} \frac{T^*}{T_e^*} u - \frac{\int_{T_e}^T c_{pa}(T) dT}{c_p(T)} \frac{1}{m} \frac{dm}{dt} \quad (3.4D)$$

The two terms on the right give the effects on temperature of adiabatic expansion and entrainment, respectively.

### 3.4.2 Wet

Since the temperature of the saturated cloud is at most  $373^\circ K$ ,

specific heats are taken as independent of temperature. As before,  $c_p$  is the weighted mean of  $c_{pa}$  and  $c_{pw}$ . For a small interval  $dt$ , the heat released by condensation:  $-mLdx/(1+x)^2$ , is equal to the sum of

Heat used to saturate and warm entrained air:

$$L(x - x_e) dm + c_{pa} (T - T_e) dm$$

and

Heat absorbed in expansion  $dp$ :

$$m \left[ c_p dT - R_a T^* \frac{dp}{p} \right]$$

This heat balance neglects the small heat release by cooling of liquid or solid water,  $-mwcdT$ , where  $c$  = specific heat of liquid or solid water. Performing the indicated balance, as for the "dry" heat equation and also using Eq. (3.3W), and dropping small terms of order  $x^2$ , and taking  $c_{pa} \approx c_p$ ,

$$\frac{dT}{dt} = - \frac{\frac{g}{c_p} \frac{T^*}{T_e} \left( 1 + \frac{xL}{R_a T} \right) u + \left[ (x - x_e) \frac{L}{c_p} + (T - T_e) \right] \frac{1}{m} \frac{dm}{dt}}{1 + \frac{L^2 \epsilon_x}{c_p R_a T^2}} \quad (3.4W)$$

The two terms on the right (corresponding to the two terms in the numerator) are respectively the ("wet") adiabatic and entrainment terms.

### 3.5 VOLUME

In a small height  $dz$ , cloud volume changes an amount  $dV$ , due to:

1. Adiabatic expansion at constant mass,  $dV_m$ , and

2. Mass entrainment at constant pressure,  $dV)_p$ , so that

$$dV = dV)_m + dV)_p$$

To calculate each  $dV$ , the gas law,  $p = \frac{m}{V} R_a q(x) T$ , is used in the differential form:

$$\frac{dp}{p} = \frac{dm}{m} + \frac{dT}{T} + \frac{dq(x)}{q(x)} - \frac{dV}{V}$$

This assumes the non-gas mass fraction is small.

### 3.5.1 Dry

$c_p(T)$  is the temperature-dependent weighted mean of  $c_{pa}$  and  $c_{pw}$ . For an adiabatic volume change at constant mass,  $dm$  and  $dq$  are zero and  $\frac{dp}{p} = \frac{\gamma}{\gamma-1} \frac{dT}{T}$ , (where  $\gamma$  is the ratio of specific heats of the cloud gas), so that

$$\frac{dV)_m}{V} = \frac{dT)_m}{T(\gamma-1)} = \frac{dT)_m}{T} \left( \frac{c_p}{R_a q(x)} - 1 \right)$$

Using the adiabatic term in Eq. (3.4D),

$$dV)_m = \frac{V_g q(x)}{T_e} \left( \frac{1}{R_a q(x)} - \frac{1}{c_p} \right)$$

For constant-pressure entrainment,

$$\frac{dV)_p}{V} = \frac{dT)_p}{T} + \frac{dm}{m} + \frac{dq(x)}{q(x)}$$

$\frac{dT}{dt}$ )<sub>p</sub> is given by the entrainment term in Eq. (3.4D), and

$$dq = \frac{1/\epsilon - 1}{(1+x)^2} dx$$

Adding the two volume changes,

$$\frac{dV}{dt} = V \left[ \frac{\rho q(x)}{T_e} \left( \frac{1}{R_a q(x)} - \frac{1}{c_p} \right) u + \left( 1 - \frac{\int_{T_e}^T c_{pa}(T) dT}{T_e - T_c p} \right) \frac{1}{m} \frac{dm}{dt} \right. \\ \left. + \frac{1/\epsilon - 1}{q(x) (1+x)^2} \frac{dx}{dt} \right] \quad (3.5D)$$

### 3.5.2 Wet

Using the adiabatic term in Eq. (3.4W),

$$(dV)_m = \frac{V g q(x)}{T_e} \left( \frac{1}{R_a q(x)} - \frac{1}{c_p} \right) \left[ \frac{1 + \frac{xL}{R_a t}}{1 + \frac{L^2 \epsilon x}{c_p R_a T^2}} \right]$$

Using the entrainment term in Eq. (3.4W),

$$(dV)_p = V \left[ 1 - \frac{(x-x_e) \frac{L}{c_p} + (T-T_e)}{T \left( 1 + \frac{L^2 \epsilon x}{c_p R_a T^2} \right)} \right] \frac{dm}{m}$$

where  $dq$  is neglected.

Adding the two volume changes,

$$\frac{dV}{dt} = \frac{Vgq(x)}{T_e} * \left( \frac{1}{R_a q(x)} - \frac{1}{c_p} \right) \left[ \frac{\frac{1 + \frac{xL}{R_a T}}{a}}{1 + \frac{L^2 \epsilon x}{c_p R_a T^2}} \right] u \quad (3.5W)$$

$$+ V \left[ 1 - \frac{(x-x_e) \frac{L}{c_p} + (T - T_e)}{T \left( 1 + \frac{L^2 \epsilon x}{c_p R_a T^2} \right)} \right] \frac{1}{m} \frac{dm}{dt}$$

### 3.6 LIQUID AND SOLID WATER CONTENT

Liquid and solid water content appear only in the wet (W) equations. If the cloud expands or warms up so much that its vapor pressure drops below saturation, then the dry equations again apply. The "wet" equations do not know about this. But if the ratio of liquid and solid water mass to dry air mass,  $w$ , drops to zero in Eq. (3.6W), a return to the "dry" equations is required.

#### 3.6.1 Dry

Let  $w$  be the ratio of liquid and solid water mass to dry air mass,  $w = m_{wi}/m_a$ . Then,

$$w = 0 \quad (3.6D)$$

#### 3.6.2 Wet

The liquid and solid water mass can increase by

1. Excess of the mixing ratio of entrained air over that of the saturated cloud, and
2. Condensation of vapor already in the cloud, so that

$$\frac{dm_{w\lambda}}{dt} = \frac{x_e - x}{1 + x_e} \frac{dm}{dt} - m_a \frac{dx}{dt}$$

By definition of  $w$ ,

$$\begin{aligned} m_a \frac{dw}{dt} &= -w \frac{dm_a}{dt} + \frac{dm_{w\lambda}}{dt} \\ &= -\left[ \frac{w + x - x_e}{1 + x_e} \right] \frac{dm}{dt} - m_a \frac{dx}{dt} \end{aligned}$$

Substituting  $m_a = \frac{m}{1 + x + w}$ ,

$$\frac{dw}{dt} = -\frac{1}{\beta} \left[ \frac{1 + x}{1 + x_e} \right] (w + x - x_e) \frac{1}{m} \frac{dm}{dt} - \frac{dx}{dt} \quad (3.6w)$$

### 3.7 TURBULENT KINETIC ENERGY DENSITY

The energy transfer rate, or the kinetic energy loss corresponding to the  $k_2$  (momentum loss) term in Eq. (3.1), is the rate of generation of turbulent energy per unit mass (Sec. 2.6.2, Eq. (2.12)). The gain of turbulent energy per unit mass is diluted by the increase of mass, so that the turbulent energy density  $E_K$  is given by

$$\frac{dE_K}{dt} = k_2 \frac{T^*}{T_e} \beta \frac{u^2 v}{m} - E_K \frac{1}{m} \frac{dm}{dt} \quad (3.7)$$

Alternatively, if  $\ell$  is independent of  $\rho/\rho_e$ ,

$$\frac{dE_k}{dt} = \frac{k_2 u^2 v}{\ell} - E_k \frac{1}{m} \frac{dm}{dt} \quad (3.7A)$$

### 3.8 MASS

As discussed in Sec. 2.6.1,

$$\frac{1}{m} \frac{dm}{dt} = \frac{S}{V} \lambda v \quad (3.8)$$

where  $S$  is cloud surface area. This is Eq. (2.2) in Sec. 2.6.1.

Alternatively, if the entrainment rate is independent of density ratio,

$$\frac{1}{m} \frac{dm}{dt} = \frac{S}{V} \frac{T^*}{T_e} \beta \lambda v \quad (3.8A)$$

This follows from Eq. (2.1) in Sec. 2.6.1.

### 3.9 CLOUD FORM: VOLUME

For a spherical cloud

$$r = \left( \frac{3V}{4\pi} \right)^{1/3} \quad (3.9)$$

where  $r$  is the radius. If a certain altitude  $z_T$  ( $T$  for tropopause) is designated such that the vertical radius remains equal to  $r(z_T) = r_T$  after  $z = z_T$ , then for the resulting oblate spheroid

$$r_h = \left( \frac{3V}{4\pi r_T} \right)^{1/2} \quad (3.9E)$$

where  $r_h$  is the horizontal radius (major semi-axis) of the spheroid.

### 3.10 CLOUD FORM: EFFECTIVE SURFACE AREA

For a spherical cloud

$$S = 4\pi r^2 \quad (3.10)$$

For an oblate spheroid cloud

$$S = 2\pi r_h^2 + \pi \frac{r_T^2}{e} \ln \left( \frac{1+e}{1-e} \right) \quad (3.10E)$$

where  $e$  is the eccentricity,

$$e = \sqrt{\frac{r_h^2 - r_T^2}{r_h^2}}$$

### 3.11 CHARACTERISTIC LENGTH

For the present we disregard compressibility effects (2.6.4) and take as characteristic length,  $\ell$

$$\ell = r \quad (3.11)$$

for the sphere, and

$$\ell = r_T \quad (3.11E)$$

for the spheroid.

### 3.12 THE EFFECT OF FREEZING

To allow for the effect of freezing, let  $L$  be the latent heat of

sublimation, when  $T$  is less than some temperature  $T_F$ . This neglects the liquid-to-solid heat release if condensation occurs both above and below  $T_F$  and the difference in vapor pressure over ice and water below  $T_F$ .

### 3.13 COMMENTS ON THE SET OF EQUATIONS

The set of equations to be solved consists of the eight differential equations (3.1) to (3.8), with the three algebraic equations (3.9) to (3.11) (or (3.9E) to (3.11E) for a spheroid) substituted in them. The initial conditions, defining equations for parameters and values of constants are given in Appendix C.

The total number of equations can be reduced by substitution. But the present extended form is more easily revised and more suitable for machine computation.

Certain equations can be divided by  $u = \frac{dz}{dt}$  to give a "quasi-static" subset with  $z$  as the independent variable. Under such restrictive assumptions as constant specific heat, zero water content and the characteristic length rule,  $\ell_1$  independent of density ratio, as in Eq. (3.1A), an analytic solution for  $r(z)$  can be obtained. These algebraic curiosities are not discussed further in this report.

### 3.14 NUMERICAL SOLUTION OF THE EQUATIONS

The eight differential equations (3.1) to (3.8) with the defining equations (3.9) to (3.11) have been programmed for machine computation, using a Runge-Kutta method. The program permits the alternative density ratio assumption (use of the "A" equations) and certain other options. Appendix E gives the FORTRAN program, using the characteristic velocity  $v = \max(|u|, \sqrt{2 E_k})$  as discussed in Sec. 2.6.3.

Tables 3.1 and 3.2 are sample computer outputs for two sets of initial conditions and parameter values. A modified tropical atmosphere<sup>23</sup> was used. Appendix D gives a glossary of printout symbols. All quantities are in mks units, except explosion energy W in kilotons.

Table 3.1 is a sample output for a 5 MT explosion with 1/3 of the energy remaining in the initial cloud, centered to just intersect sea level. Note the late horizontal expansion of the cloud (column R).

Computations were also made for the same initial conditions and parameters as those for Table 3.1, but using the alternate equations (3.1A), (3.7A), and (3.8A), omitting the initial-virtual-mass factor, and with  $k_2 = 0$ , i.e. no eddy-viscous force and thus no turbulence effects. This is essentially the original Taylor<sup>4</sup> hypothesis except for the sphere-spheroid transition at the tropopause. It was found that this hypothesis did not account for the late horizontal expansion of the cloud, and predicted much larger vertical oscillations of the cloud, after the cloud reached maximum height.

Table 3.2 is the output for a low yield (20 KT) shot. It shows the much lower turbulent energy density (column EK) of low yield clouds. Note also that the cloud does not reach a maximum height at about 360 sec., but continues rising slowly. This is an effect of the humid tropical atmosphere ("conditional instability") and agrees with Pacific test observations.

The numerical results are in general agreement with observations of atomic cloud radius, rate of rise and (for megaton clouds) late horizontal expansion, when  $k_2$  is about 0.10 to 0.15 and  $\lambda$  about 0.20 to 0.30 (and when the initial virtual mass factor is included in the momentum equation). This indicates that (as suggested in Sec. 2.6.2)  $2k_2$  is closely related to  $\lambda$ , so that  $2k_2$  may be replaced by  $\lambda$ .

DST= 0.06250 K2= 0.1250000 LAMBDA= 0.250000 C1= 0.8000000 C2= 0.000040  
 TEO=300.000000 CHANGE= 30.000 DST2= 5.0000 80= 1091.00 81= 0.  
 A1= 0.00650 A2=-0.00440 A3=-0.00220 A4= 0. Z1= 16500. Z2= 22000  
 ZT= 16500. DO= 1910.0 D1= 0.03490 D2=0.00031300 TF= 273.0 C3=0.

ST	U	X	T	R	Z	EK	V	WT	TE
0.	0.	0.3329	3000.0	1540.	1500.	0.	0.153E 11	0.	300
1.0	15.7	0.3313	2993.8	1541.	1508.	1.	0.153E 11	0.	290
2.0	31.2	0.3265	2975.2	1544.	1531.	14.	0.154E 11	0.	290
3.0	46.4	0.3188	2944.4	1550.	1570.	71.	0.156E 11	0.	289
4.0	61.2	0.3085	2902.0	1557.	1624.	215.	0.158E 11	0.	289
5.0	75.4	0.2962	2848.3	1567.	1692.	494.	0.161E 11	0.	289
6.0	88.9	0.2822	2784.2	1578.	1775.	956.	0.165E 11	0.	288
7.0	101.7	0.2671	2710.5	1591.	1870.	1635.	0.169E 11	0.	287
8.0	113.5	0.2513	2628.4	1606.	1978.	2547.	0.173E 11	0.	287
9.0	124.5	0.2353	2539.0	1621.	2097.	3689.	0.179E 11	0.	286
10.0	134.5	0.2194	2443.7	1638.	2226.	5040.	0.184E 11	0.	285
11.0	143.5	0.2039	2343.9	1656.	2365.	6562.	0.190E 11	0.	284
12.0	151.6	0.1890	2241.1	1675.	2513.	8206.	0.197E 11	0.	283
13.0	158.7	0.1749	2136.6	1694.	2668.	9917.	0.204E 11	0.	282
14.0	164.9	0.1615	2031.9	1714.	2830.	11640.	0.211E 11	0.	281
15.0	170.3	0.1491	1928.2	1734.	2998.	13324.	0.218E 11	0.	280
16.0	174.8	0.1375	1826.5	1755.	3170.	14924.	0.226E 11	0.	279
17.0	178.5	0.1268	1727.5	1777.	3347.	16410.	0.235E 11	0.	278
18.0	181.2	0.1167	1630.2	1799.	3527.	17773.	0.244E 11	0.	277
19.0	183.0	0.1073	1535.7	1821.	3709.	18975.	0.253E 11	0.	275
20.0	184.0	0.0985	1444.7	1845.	3893.	19989.	0.263E 11	0.	274
21.0	184.3	0.0905	1358.1	1869.	4077.	20801.	0.273E 11	0.	273
22.0	184.0	0.0832	1276.4	1893.	4261.	21411.	0.284E 11	0.	272
23.0	183.2	0.0765	1199.9	1919.	4445.	21827.	0.296E 11	0.	271
24.0	182.0	0.0704	1128.7	1944.	4628.	22064.	0.308E 11	0.	269
25.0	180.5	0.0648	1062.7	1971.	4809.	22143.	0.321E 11	0.	268
26.0	178.7	0.0598	1001.8	1998.	4989.	22085.	0.334E 11	0.	267
27.0	176.7	0.0553	945.9	2025.	5166.	21913.	0.348E 11	0.	266
28.0	174.6	0.0512	894.5	2053.	5342.	21647.	0.363E 11	0.	265
29.0	172.4	0.0475	847.5	2082.	5515.	21307.	0.378E 11	0.	264
30.0	170.1	0.0441	804.5	2111.	5687.	20909.	0.394E 11	0.	263
35.0	158.6	0.0315	638.4	2265.	6509.	18504.	0.486E 11	0.	257
40.0	147.9	0.0235	530.4	2429.	7274.	16064.	0.600E 11	0.	252
45.0	138.6	0.0183	457.7	2601.	7990.	13950.	0.737E 11	0.	248
50.0	130.6	0.0146	406.8	2779.	8662.	12211.	0.899E 11	0.	243
55.0	123.8	0.0120	369.7	2961.	9298.	10798.	0.109E 12	0.	239
60.0	117.9	0.0101	341.6	3146.	9902.	9648.	0.130E 12	0.	235
65.0	112.8	0.0086	319.6	3334.	10478.	8705.	0.155E 12	0.	231
70.0	108.3	0.0075	301.9	3523.	11031.	7923.	0.183E 12	0.	228
75.0	104.3	0.0066	287.3	3713.	11562.	7268.	0.214E 12	0.	224
80.0	100.7	0.0059	275.0	3904.	12075.	6712.	0.249E 12	0.	221
85.0	97.5	0.0053	264.5	4096.	12570.	6237.	0.288E 12	0.	218
90.0	94.5	0.0048	255.3	4289.	13050.	5826.	0.331E 12	0.	215
95.0	91.7	0.0044	247.1	4483.	13515.	5468.	0.377E 12	0.	212
90.5	94.2	0.0047	254.4	4309.	13097.	5788.	0.335E 12	0.	214
91.0	93.9	0.0047	253.6	4328.	13144.	5751.	0.340E 12	0.	214
91.5	93.6	0.0046	252.7	4347.	13191.	5714.	0.344E 12	0.	214



00 LAM80A= 0.250000 C1= 0.8000000 C2= 0.0000400 W=0.500000E 04 F= 0.33333333 PHI= 0.50000000  
 30.000 DST2= 5.0000 80= 1091.00 81= 0.1328 82= 0. DST1= 1.0 K=2  
 0 A3=-0.00220 A4= 0. Z1= 16500. Z2= 22000. Z3= 52000. P0= 101300.  
 0 D1= 0.03490 D2=0.00031300 TF= 273.0 C3=0. PRINT= 900.0 RK3= 1.0

2

T	R	Z	EK	V	WT	TE	M	ES	P	PW	EO
000.0	1540.	1500.	0.	0.153E 11	0.	300.0	0.131E 10	20666449.	85159.	29728.	0.
993.8	1541.	1508.	1.	0.153E 11	0.	290.2	0.132E 10	20789247.	85080.	29606.	3.691
975.2	1544.	1531.	14.	0.154E 11	0.	290.0	0.133E 10	21160812.	84846.	29246.	28.858
944.4	1550.	1570.	71.	0.156E 11	0.	289.8	0.136E 10	21790562.	84459.	28659.	93.575
902.0	1557.	1624.	215.	0.158E 11	0.	289.4	0.139E 10	22693952.	83924.	27866.	209.868
848.3	1567.	1692.	494.	0.161E 11	0.	289.0	0.144E 10	23892050.	83250.	26892.	382.061
784.2	1578.	1775.	956.	0.165E 11	0.	288.5	0.150E 10	25410634.	82444.	25767.	606.515
710.5	1591.	1870.	1635.	0.169E 11	0.	287.8	0.157E 10	27278779.	81518.	24524.	872.629
628.4	1606.	1978.	2547.	0.173E 11	0.	287.1	0.165E 10	29526407.	80482.	23196.	1164.822
539.0	1621.	2097.	3689.	0.179E 11	0.	286.4	0.174E 10	32180835.	79349.	21814.	1465.010
443.7	1638.	2226.	5040.	0.184E 11	0.	285.5	0.185E 10	35261790.	78131.	20408.	1755.113
343.9	1656.	2365.	6562.	0.190E 11	0.	284.6	0.197E 10	38775054.	76840.	19004.	2019.158
241.1	1675.	2513.	8206.	0.197E 11	0.	283.7	0.211E 10	42704862.	75489.	17624.	2244.737
136.6	1694.	2668.	9917.	0.204E 11	0.	282.7	0.226E 10	47005723.	74089.	16286.	2423.723
031.9	1714.	2830.	11640.	0.211E 11	0.	281.6	0.242E 10	51594862.	72651.	15003.	2552.302
928.2	1734.	2998.	13324.	0.218E 11	0.	280.5	0.261E 10	56346872.	71186.	13786.	2630.492
826.5	1755.	3170.	14924.	0.226E 11	0.	279.4	0.281E 10	61092443.	69704.	12641.	2661.354
727.5	1777.	3347.	16410.	0.235E 11	0.	278.2	0.303E 10	65637857.	68211.	11569.	2687.321
630.2	1799.	3527.	17773.	0.244E 11	0.	277.1	0.327E 10	69806646.	66718.	10558.	2685.909
535.7	1821.	3709.	18975.	0.253E 11	0.	275.9	0.354E 10	73294519.	65234.	9613.	2633.862
444.7	1845.	3893.	19989.	0.263E 11	0.	274.7	0.384E 10	75784028.	63765.	8737.	2539.523
358.1	1869.	4077.	20801.	0.273E 11	0.	273.5	0.417E 10	76996588.	62318.	7931.	2413.414
276.4	1893.	4261.	21411.	0.284E 11	0.	272.3	0.452E 10	76737113.	60898.	7195.	2265.949
199.9	1919.	4445.	21827.	0.296E 11	0.	271.1	0.491E 10	74928842.	59508.	6526.	2106.454
128.7	1944.	4628.	22064.	0.308E 11	0.	269.9	0.532E 10	71629725.	58152.	5920.	1942.640
062.7	1971.	4809.	22143.	0.321E 11	0.	268.7	0.577E 10	67025669.	56830.	5374.	1780.450
001.8	1998.	4989.	22085.	0.334E 11	0.	267.6	0.625E 10	61402925.	55545.	4883.	1624.142
945.9	2025.	5166.	21913.	0.348E 11	0.	266.4	0.676E 10	55106145.	54297.	4441.	1476.527
894.5	2053.	5342.	21647.	0.363E 11	0.	265.3	0.729E 10	48492525.	53085.	4045.	1339.247
847.5	2082.	5515.	21307.	0.378E 11	0.	264.1	0.786E 10	41891154.	51910.	3689.	1213.058
804.5	2111.	5687.	20909.	0.394E 11	0.	263.0	0.846E 10	35573844.	50771.	3371.	1098.084
638.4	2265.	6509.	18504.	0.486E 11	0.	257.7	0.119E 11	12814587.	45600.	2202.	682.996
530.4	2429.	7274.	16064.	0.600E 11	0.	252.7	0.160E 11	3760633.	41150.	1502.	432.792
457.7	2601.	7990.	13950.	0.737E 11	0.	248.1	0.207E 11	1038567.	37320.	1066.	288.990
406.8	2779.	8662.	12211.	0.899E 11	0.	243.7	0.260E 11	294205.	33991.	783.	202.580
369.7	2961.	9298.	10798.	0.109E 12	0.	239.6	0.317E 11	89100.	31071.	591.	148.068
341.6	3146.	9902.	9648.	0.130E 12	0.	235.6	0.378E 11	29260.	28488.	457.	112.073
319.6	3334.	10478.	8705.	0.155E 12	0.	231.9	0.442E 11	10417.	26187.	360.	87.333
301.9	3523.	11031.	7923.	0.183E 12	0.	228.3	0.509E 11	3994.	24126.	288.	69.691
287.3	3713.	11562.	7268.	0.214E 12	0.	224.8	0.578E 11	1635.	22269.	234.	56.726
275.0	3904.	12075.	6712.	0.249E 12	0.	221.5	0.649E 11	708.	20589.	193.	46.942
264.5	4096.	12570.	6237.	0.288E 12	0.	218.3	0.722E 11	321.	19065.	160.	39.384
255.3	4289.	13050.	5826.	0.331E 12	0.	215.2	0.797E 11	152.	17677.	135.	33.428
247.1	4483.	13515.	5468.	0.377E 12	0.	212.1	0.873E 11	74.	16410.	114.	28.651
254.4	4309.	13097.	5788.	0.335E 12	0.	214.9	0.804E 11	141.	17545.	133.	32.887
253.6	4328.	13144.	5751.	0.340E 12	0.	214.6	0.812E 11	131.	17414.	130.	32.374
252.7	4347.	13191.	5714.	0.344E 12	0.	214.3	0.819E 11	122.	17285.	128.	31.872

ST	U	X	T	R	Z	EK	V	WT	TE
<b>SWITCH TO WET</b>									
96.5	91.1	0.0031	247.7	4515.	13653.	5369.	0.385E 12	0.0012	21
101.5	89.1	0.0019	242.5	4686.	14103.	5066.	0.431E 12	0.0020	20
106.5	87.4	0.0012	237.1	4861.	14544.	4799.	0.481E 12	0.0024	20
111.5	85.9	0.0007	231.6	5043.	14978.	4561.	0.537E 12	0.0027	20
116.5	84.5	0.0004	226.1	5229.	15404.	4349.	0.599E 12	0.0027	19
121.5	83.1	0.0002	220.8	5421.	15823.	4158.	0.667E 12	0.0027	19
126.5	81.7	0.0001	215.6	5617.	16235.	3986.	0.742E 12	0.0026	19
131.5	80.2	0.0001	210.7	5816.	16640.	3829.	0.824E 12	0.0025	19
<b>SWITCH TO ELLIPSE, R=RH</b>									
136.5	77.9	0.0000	206.2	6121.	17035.	3685.	0.913E 12	0.0024	19
141.5	74.5	0.0000	202.3	6431.	17417.	3551.	0.101E 13	0.0023	19
146.5	70.4	0.0000	199.0	6745.	17779.	3426.	0.111E 13	0.0022	19
151.5	65.7	0.0000	196.1	7059.	18120.	3307.	0.121E 13	0.0021	19
156.5	60.5	0.0000	193.7	7371.	18436.	3192.	0.132E 13	0.0020	20
161.5	54.9	0.0000	191.7	7679.	18724.	3083.	0.144E 13	0.0019	20
166.5	49.1	0.0000	190.0	7981.	18985.	2977.	0.155E 13	0.0019	20
171.5	43.1	0.0000	188.8	8273.	19215.	2876.	0.167E 13	0.0018	20
176.5	37.1	0.0000	187.8	8556.	19415.	2778.	0.178E 13	0.0017	20
181.5	31.0	0.0000	187.1	8826.	19586.	2683.	0.190E 13	0.0017	20
186.5	25.0	0.0000	186.7	9083.	19726.	2593.	0.201E 13	0.0016	20
191.5	19.2	0.0000	186.6	9325.	19836.	2507.	0.212E 13	0.0015	20
196.5	13.6	0.0000	186.7	9552.	19918.	2424.	0.222E 13	0.0015	20
201.5	8.2	0.0000	186.9	9763.	19972.	2345.	0.232E 13	0.0014	20
206.5	3.1	0.0000	187.4	9958.	20000.	2271.	0.242E 13	0.0014	20
211.5	-1.7	0.0000	188.0	10136.	20003.	2200.	0.250E 13	0.0013	20
216.5	-6.1	0.0000	188.8	10299.	19984.	2133.	0.258E 13	0.0013	20
221.5	-10.2	0.0000	189.7	10445.	19943.	2070.	0.266E 13	0.0013	20
226.5	-13.8	0.0000	190.7	10577.	19883.	2010.	0.273E 13	0.0012	20
231.5	-16.9	0.0000	191.8	10695.	19806.	1954.	0.279E 13	0.0012	20
236.5	-19.7	0.0000	192.9	10800.	19714.	1901.	0.284E 13	0.0012	20
241.5	-21.9	0.0000	194.2	10893.	19610.	1851.	0.289E 13	0.0011	20
246.5	-23.7	0.0000	195.4	10976.	19495.	1804.	0.293E 13	0.0011	20
251.5	-25.1	0.0000	196.7	11050.	19373.	1759.	0.297E 13	0.0011	20
256.5	-26.0	0.0000	198.0	11117.	19245.	1717.	0.301E 13	0.0010	20
261.5	-26.4	0.0000	199.2	11178.	19114.	1676.	0.304E 13	0.0010	20
266.5	-26.5	0.0000	200.5	11235.	18982.	1637.	0.308E 13	0.0010	20
271.5	-26.1	0.0000	201.7	11289.	18850.	1600.	0.310E 13	0.0009	20
276.5	-25.3	0.0000	202.8	11343.	18721.	1564.	0.313E 13	0.0009	20
281.5	-24.2	0.0000	203.8	11396.	18598.	1529.	0.316E 13	0.0009	20
286.5	-22.8	0.0000	204.8	11452.	18480.	1495.	0.319E 13	0.0009	20
291.5	-21.1	0.0000	205.7	11510.	18370.	1462.	0.323E 13	0.0008	20
296.5	-19.1	0.0000	206.4	11573.	18270.	1430.	0.326E 13	0.0008	20
301.5	-17.0	0.0001	207.1	11641.	18179.	1398.	0.330E 13	0.0008	20
306.5	-14.7	0.0001	207.6	11714.	18100.	1367.	0.334E 13	0.0008	19
311.5	-12.3	0.0001	208.0	11795.	18033.	1337.	0.339E 13	0.0007	19
316.5	-9.8	0.0001	208.3	11883.	17977.	1308.	0.344E 13	0.0007	19
321.5	-7.3	0.0001	208.5	11979.	17935.	1279.	0.350E 13	0.0007	19
326.5	-4.8	0.0001	208.6	12083.	17904.	1251.	0.356E 13	0.0007	19
331.5	-2.4	0.0001	208.5	12195.	17886.	1225.	0.362E 13	0.0007	19
336.5	-0.0	0.0001	208.4	12316.	17880.	1199.	0.370E 13	0.0007	19
341.5	2.2	0.0001	208.1	12445.	17886.	1174.	0.377E 13	0.0006	19
346.5	4.3	0.0001	207.8	12583.	17902.	1149.	0.386E 13	0.0006	19
351.5	6.2	0.0001	207.4	12728.	17928.	1126.	0.395E 13	0.0006	19
356.5	7.9	0.0000	206.9	12881.	17963.	1104.	0.404E 13	0.0006	19
361.5	9.4	0.0000	206.4	13041.	18007.	1082.	0.414E 13	0.0006	19
366.5	10.7	0.0000	205.8	13207.	18057.	1062.	0.425E 13	0.0006	19



R	Z	EK	V	WT	TE	M	ES	P	PW	EC	
7.7	4515.	13653.	5369.	0.385E 12	0.0012	211.3	0.896E 11	79.	16050.	79.	27.958
2.5	4686.	14103.	5066.	0.431E 12	0.0020	208.3	0.973E 11	48.	14915.	46.	24.808
7.1	4861.	14544.	4799.	0.481E 12	0.0024	205.5	0.105E 12	29.	13868.	26.	22.182
1.6	5043.	14978.	4561.	0.537E 12	0.0027	202.6	0.113E 12	16.	12898.	14.	19.931
6.1	5229.	15404.	4349.	0.599E 12	0.0027	199.9	0.121E 12	9.	11999.	7.	17.964
0.8	5421.	15823.	4158.	0.667E 12	0.0027	197.2	0.129E 12	5.	11164.	4.	16.223
5.6	5617.	16235.	3986.	0.742E 12	0.0026	194.5	0.138E 12	3.	10390.	2.	14.671
0.7	5816.	16640.	3829.	0.824E 12	0.0025	193.4	0.146E 12	1.	9673.	1.	13.175
6.2	6121.	17035.	3685.	0.913E 12	0.0024	195.1	0.154E 12	1.	9024.	1.	11.799
2.3	6431.	17417.	3551.	0.101E 13	0.0023	196.8	0.162E 12	0.	8444.	0.	10.328
9.0	6745.	17779.	3426.	0.111E 13	0.0022	198.4	0.171E 12	0.	7931.	0.	8.837
6.1	7059.	18120.	3307.	0.121E 13	0.0021	199.9	0.179E 12	0.	7481.	0.	7.391
3.7	7371.	18436.	3192.	0.132E 13	0.0020	201.3	0.187E 12	0.	7089.	0.	6.039
1.7	7679.	18724.	3083.	0.144E 13	0.0019	202.5	0.196E 12	0.	6751.	0.	4.810
0.0	7981.	18985.	2977.	0.155E 13	0.0019	203.7	0.204E 12	0.	6462.	0.	3.724
8.8	8273.	19215.	2876.	0.167E 13	0.0018	204.7	0.212E 12	0.	6218.	0.	2.790
7.8	8556.	19415.	2778.	0.178E 13	0.0017	205.6	0.221E 12	0.	6014.	0.	2.007
7.1	8826.	19586.	2683.	0.190E 13	0.0017	206.3	0.229E 12	0.	5847.	0.	1.371
6.7	9083.	19726.	2593.	0.201E 13	0.0016	206.9	0.238E 12	0.	5713.	0.	0.874
6.6	9325.	19836.	2507.	0.212E 13	0.0015	207.4	0.246E 12	0.	5610.	0.	0.504
6.7	9552.	19918.	2424.	0.222E 13	0.0015	207.8	0.255E 12	0.	5535.	0.	0.247
6.9	9763.	19972.	2345.	0.232E 13	0.0014	208.0	0.263E 12	0.	5486.	0.	0.088
7.4	9958.	20000.	2271.	0.242E 13	0.0014	208.2	0.272E 12	0.	5460.	0.	0.012
8.0	10136.	20003.	2200.	0.250E 13	0.0013	208.2	0.281E 12	0.	5458.	0.	0.004
8.8	10299.	19984.	2133.	0.258E 13	0.0013	208.1	0.290E 12	0.	5475.	0.	0.048
9.7	10445.	19943.	2070.	0.266E 13	0.0013	207.9	0.299E 12	0.	5512.	0.	0.130
9.0	10577.	19883.	2010.	0.273E 13	0.0012	207.6	0.308E 12	0.	5567.	0.	0.237
9.1	10695.	19806.	1954.	0.279E 13	0.0012	207.3	0.317E 12	0.	5638.	0.	0.356
9.2	10800.	19714.	1901.	0.284E 13	0.0012	206.9	0.326E 12	0.	5724.	0.	0.478
9.2	10893.	19610.	1851.	0.289E 13	0.0011	206.4	0.335E 12	0.	5823.	0.	0.591
9.5	10976.	19495.	1804.	0.293E 13	0.0011	205.9	0.345E 12	0.	5934.	0.	0.690
9.6	11050.	19373.	1759.	0.297E 13	0.0011	205.4	0.354E 12	0.	6056.	0.	0.768
9.8	11117.	19245.	1717.	0.301E 13	0.0010	204.8	0.364E 12	0.	6186.	0.	0.821
9.9	11178.	19114.	1676.	0.304E 13	0.0010	204.3	0.373E 12	0.	6323.	0.	0.848
0.5	11235.	18982.	1637.	0.308E 13	0.0010	203.7	0.383E 12	0.	6465.	0.	0.847
0.1	11289.	18850.	1600.	0.310E 13	0.0009	203.1	0.393E 12	0.	6610.	0.	0.820
0.2	11343.	18721.	1564.	0.313E 13	0.0009	202.5	0.403E 12	0.	6754.	0.	0.771
0.3	11396.	18598.	1529.	0.316E 13	0.0009	202.0	0.413E 12	1.	6897.	0.	0.702
0.4	11452.	18480.	1495.	0.319E 13	0.0009	201.5	0.424E 12	1.	7036.	0.	0.619
0.5	11510.	18370.	1462.	0.323E 13	0.0008	201.0	0.434E 12	1.	7168.	1.	0.528
0.6	11573.	18270.	1430.	0.326E 13	0.0008	200.5	0.445E 12	1.	7292.	1.	0.433
0.7	11641.	18179.	1398.	0.330E 13	0.0008	200.1	0.455E 12	1.	7405.	1.	0.339
0.7	11714.	18100.	1367.	0.334E 13	0.0008	199.8	0.466E 12	1.	7506.	1.	0.252
0.8	11795.	18033.	1337.	0.339E 13	0.0007	199.5	0.477E 12	1.	7593.	1.	0.175
0.8	11883.	17977.	1308.	0.344E 13	0.0007	199.3	0.488E 12	1.	7665.	1.	0.111
0.8	11979.	17935.	1279.	0.350E 13	0.0007	199.1	0.499E 12	1.	7722.	1.	0.061
0.8	12083.	17904.	1251.	0.356E 13	0.0007	198.9	0.510E 12	1.	7762.	1.	0.026
0.8	12195.	17886.	1225.	0.362E 13	0.0007	198.8	0.521E 12	1.	7786.	1.	0.006
0.8	12316.	17880.	1199.	0.370E 13	0.0007	198.8	0.533E 12	1.	7794.	1.	0.000
0.8	12445.	17886.	1174.	0.377E 13	0.0006	198.8	0.544E 12	1.	7787.	1.	0.005
0.7	12583.	17902.	1149.	0.386E 13	0.0006	198.9	0.555E 12	1.	7765.	1.	0.020
0.7	12728.	17928.	1126.	0.395E 13	0.0006	199.0	0.567E 12	1.	7730.	1.	0.040
0.6	12881.	17963.	1104.	0.404E 13	0.0006	199.2	0.579E 12	1.	7684.	1.	0.065
0.6	13041.	18007.	1082.	0.414E 13	0.0006	199.4	0.590E 12	1.	7627.	1.	0.091
0.5	13207.	18057.	1062.	0.425E 13	0.0006	199.6	0.602E 12	1.	7562.	1.	0.116

ST	U	X	T	R	Z	EK	V	WT	TE
371.5	11.7	0.0000	205.2	13379.	18113	1042.	0.436E 13	0.0006	199.8
376.5	12.5	0.0000	204.6	13556.	18177.	1023.	0.448E 13	0.0006	200.1
381.5	13.1	0.0000	203.9	13737.	18238.	1005.	0.460E 13	0.0006	200.4
386.5	13.4	0.0000	203.3	13922.	18304.	987.	0.472E 13	0.0006	200.7
391.5	13.5	0.0000	202.6	14108.	18371.	970.	0.485E 13	0.0005	201.0
396.5	13.4	0.0000	202.0	14296.	18439.	953.	0.498E 13	0.0005	201.3
401.5	13.1	0.0000	201.4	14485.	18505.	937.	0.511E 13	0.0005	201.6
406.5	12.6	0.0000	200.9	14672.	18570.	922.	0.524E 13	0.0005	201.9
411.5	11.9	0.0000	200.4	14859.	18631.	906.	0.538E 13	0.0005	202.1
416.5	11.1	0.0000	199.9	15043.	18689.	891.	0.551E 13	0.0005	202.4
421.5	10.1	0.0000	199.5	15224.	18742.	877.	0.565E 13	0.0005	202.6
426.5	9.1	0.0000	199.2	15400.	18790.	863.	0.578E 13	0.0005	202.8
431.5	7.9	0.0000	198.9	15573.	18832.	849.	0.591E 13	0.0005	203.0
436.5	6.6	0.0000	198.6	15740.	18869.	835.	0.604E 13	0.0005	203.2
441.5	5.3	0.0000	198.4	15901.	18899.	822.	0.616E 13	0.0005	203.3
446.5	4.0	0.0000	198.3	16056.	18922.	809.	0.628E 13	0.0005	203.4
451.5	2.7	0.0000	198.3	16205.	18939.	796.	0.640E 13	0.0005	203.5
456.5	1.4	0.0000	198.2	16347.	18949.	783.	0.651E 13	0.0004	203.5
461.5	0.1	0.0000	198.3	16482.	18952.	771.	0.662E 13	0.0004	203.5
466.5	-1.2	0.0000	198.4	16610.	18950.	759.	0.672E 13	0.0004	203.5
471.5	-2.4	0.0000	198.6	16732.	18941.	748.	0.682E 13	0.0004	203.5
476.5	-3.5	0.0000	198.7	16847.	18926.	737.	0.691E 13	0.0004	203.4
481.5	-4.5	0.0000	199.0	16956.	18906.	726.	0.700E 13	0.0004	203.3
486.5	-5.4	0.0000	199.3	17059.	18882.	715.	0.709E 13	0.0004	203.2
491.5	-6.2	0.0000	199.6	17157.	18853.	705.	0.717E 13	0.0004	203.1
496.5	-6.8	0.0000	199.9	17250.	18820.	695.	0.725E 13	0.0004	203.0
501.5	-7.4	0.0000	200.2	17339.	18785.	685.	0.732E 13	0.0004	202.8
506.5	-7.8	0.0000	200.6	17424.	18747.	675.	0.740E 13	0.0004	202.6
511.5	-8.0	0.0000	201.0	17507.	18707.	666.	0.747E 13	0.0004	202.5
516.5	-8.2	0.0000	201.3	17587.	18667.	657.	0.754E 13	0.0004	202.3
521.5	-8.2	0.0000	201.7	17666.	18626.	648.	0.760E 13	0.0004	202.1
526.5	-8.1	0.0000	202.0	17745.	18585.	639.	0.767E 13	0.0003	201.9
531.5	-7.8	0.0000	202.4	17823.	18545.	631.	0.774E 13	0.0003	201.8
536.5	-7.5	0.0000	202.7	17902.	18507.	623.	0.781E 13	0.0003	201.6
541.5	-7.0	0.0000	203.0	17983.	18471.	614.	0.788E 13	0.0003	201.4
546.5	-6.4	0.0000	203.3	18066.	18437.	606.	0.795E 13	0.0003	201.3
551.5	-5.8	0.0000	203.5	18151.	18407.	598.	0.803E 13	0.0003	201.1
556.5	-5.1	0.0000	203.7	18239.	18380.	591.	0.810E 13	0.0003	201.0
561.5	-4.3	0.0000	203.9	18331.	18356.	583.	0.819E 13	0.0003	200.9
566.5	-3.5	0.0000	204.0	18427.	18336.	575.	0.827E 13	0.0003	200.8
571.5	-2.7	0.0000	204.1	18527.	18321.	568.	0.836E 13	0.0003	200.8
576.5	-1.8	0.0000	204.2	18632.	18310.	561.	0.846E 13	0.0003	200.7
581.5	-1.0	0.0000	204.2	18741.	18303.	554.	0.856E 13	0.0003	200.7
586.5	-0.1	0.0000	204.2	18855.	18300.	547.	0.866E 13	0.0003	200.7
591.5	0.7	0.0000	204.1	18973.	18302.	540.	0.877E 13	0.0003	200.7
596.5	1.5	0.0000	204.0	19097.	18307.	533.	0.888E 13	0.0003	200.7
601.5	2.2	0.0000	203.9	19225.	18316.	527.	0.900E 13	0.0003	200.7
606.5	2.9	0.0000	203.7	19357.	18329.	520.	0.913E 13	0.0003	200.8
611.5	3.5	0.0000	203.6	19493.	18345.	514.	0.926E 13	0.0003	200.9
616.5	4.0	0.0000	203.4	19633.	18364.	508.	0.939E 13	0.0003	201.0
621.5	4.5	0.0000	203.1	19776.	18386.	502.	0.953E 13	0.0003	201.0
626.5	4.9	0.0000	202.9	19923.	18409.	496.	0.967E 13	0.0003	201.1
631.5	5.1	0.0000	202.7	20071.	18434.	490.	0.981E 13	0.0003	201.3
636.5	5.3	0.0000	202.4	20222.	18460.	485.	0.996E 13	0.0003	201.4
641.5	5.4	0.0000	202.2	20374.	18487.	479.	0.101E 14	0.0003	201.5
646.5	5.4	0.0000	201.9	20526.	18514.	474.	0.103E 14	0.0003	201.6
651.5	5.4	0.0000	201.7	20679.	18542.	468.	0.104E 14	0.0002	201.7
656.5	5.2	0.0000	201.5	20832.	18568.	463.	0.106E 14	0.0002	201.8



R	Z	EK	V	WT	TE	M	ES	P	PW	ED
13379.	18113.	1042.	0.436E 13	0.0006	199.8	0.614E 12	1.	7490.	0.	0.138
13556.	18173.	1023.	0.448E 13	0.0006	200.1	0.626E 12	1.	7413.	0.	0.155
13737.	18238.	1005.	0.460E 13	0.0006	200.4	0.638E 12	1.	7332.	0.	0.168
13922.	18304.	987.	0.472E 13	0.0006	200.7	0.650E 12	1.	7250.	0.	0.174
14108.	18371.	970.	0.485E 13	0.0005	201.0	0.662E 12	0.	7167.	0.	0.175
14296.	18439.	953.	0.498E 13	0.0005	201.3	0.674E 12	0.	7085.	0.	0.170
14485.	18505.	937.	0.511E 13	0.0005	201.6	0.686E 12	0.	7006.	0.	0.160
14672.	18570.	922.	0.524E 13	0.0005	201.9	0.698E 12	0.	6930.	0.	0.146
14859.	18631.	906.	0.538E 13	0.0005	202.1	0.710E 12	0.	6859.	0.	0.129
15043.	18689.	891.	0.551E 13	0.0005	202.4	0.723E 12	0.	6792.	0.	0.111
15224.	18742.	877.	0.565E 13	0.0005	202.6	0.735E 12	0.	6731.	0.	0.091
15400.	18790.	863.	0.578E 13	0.0005	202.8	0.748E 12	0.	6677.	0.	0.072
15573.	18832.	849.	0.591E 13	0.0005	203.0	0.760E 12	0.	6630.	0.	0.054
15740.	18869.	835.	0.604E 13	0.0005	203.2	0.773E 12	0.	6589.	0.	0.038
15901.	18899.	822.	0.616E 13	0.0005	203.3	0.786E 12	0.	6556.	0.	0.024
16056.	18922.	809.	0.628E 13	0.0005	203.4	0.798E 12	0.	6530.	0.	0.014
16205.	18939.	796.	0.640E 13	0.0005	203.5	0.811E 12	0.	6512.	0.	0.006
16347.	18949.	783.	0.651E 13	0.0004	203.5	0.824E 12	0.	6501.	0.	0.002
16482.	18952.	771.	0.662E 13	0.0004	203.5	0.837E 12	0.	6497.	0.	0.000
16610.	18950.	759.	0.672E 13	0.0004	203.5	0.850E 12	0.	6500.	0.	0.001
16732.	18941.	748.	0.682E 13	0.0004	203.5	0.863E 12	0.	6510.	0.	0.005
16847.	18926.	737.	0.691E 13	0.0004	203.4	0.876E 12	0.	6526.	0.	0.010
16956.	18906.	726.	0.700E 13	0.0004	203.3	0.890E 12	0.	6548.	0.	0.016
17059.	18882.	715.	0.709E 13	0.0004	203.2	0.903E 12	0.	6575.	0.	0.023
17157.	18853.	705.	0.717E 13	0.0004	203.1	0.916E 12	0.	6607.	0.	0.030
17250.	18820.	695.	0.725E 13	0.0004	203.0	0.930E 12	0.	6643.	0.	0.037
17339.	18785.	685.	0.732E 13	0.0004	202.8	0.943E 12	0.	6683.	0.	0.043
17424.	18747.	675.	0.740E 13	0.0004	202.6	0.957E 12	0.	6725.	0.	0.047
17507.	18707.	666.	0.747E 13	0.0004	202.5	0.971E 12	0.	6771.	0.	0.050
17587.	18667.	657.	0.754E 13	0.0004	202.3	0.985E 12	0.	6817.	0.	0.052
17666.	18626.	648.	0.760E 13	0.0004	202.1	0.999E 12	0.	6864.	0.	0.052
17745.	18585.	639.	0.767E 13	0.0003	201.9	0.101E 13	0.	6912.	0.	0.050
17823.	18545.	631.	0.774E 13	0.0003	201.8	0.103E 13	0.	6958.	0.	0.047
17902.	18507.	623.	0.781E 13	0.0003	201.6	0.104E 13	0.	7003.	0.	0.042
17983.	18471.	614.	0.788E 13	0.0003	201.4	0.106E 13	1.	7047.	0.	0.037
18066.	18437.	606.	0.795E 13	0.0003	201.3	0.107E 13	1.	7087.	0.	0.031
18151.	18407.	598.	0.803E 13	0.0003	201.1	0.108E 13	1.	7124.	0.	0.025
18239.	18380.	591.	0.810E 13	0.0003	201.0	0.110E 13	1.	7157.	0.	0.019
18331.	18356.	583.	0.819E 13	0.0003	200.9	0.111E 13	1.	7186.	0.	0.014
18427.	18336.	575.	0.827E 13	0.0003	200.8	0.113E 13	1.	7210.	0.	0.009
18527.	18321.	568.	0.836E 13	0.0003	200.8	0.114E 13	1.	7228.	0.	0.005
18632.	18310.	561.	0.846E 13	0.0003	200.7	0.116E 13	1.	7242.	0.	0.002
18741.	18303.	554.	0.856E 13	0.0003	200.7	0.117E 13	1.	7251.	0.	0.001
18855.	18300.	547.	0.866E 13	0.0003	200.7	0.119E 13	1.	7254.	0.	0.000
18973.	18302.	540.	0.877E 13	0.0003	200.7	0.120E 13	1.	7252.	0.	0.000
19097.	18307.	533.	0.888E 13	0.0003	200.7	0.122E 13	1.	7246.	0.	0.002
19225.	18316.	527.	0.900E 13	0.0003	200.7	0.123E 13	1.	7234.	0.	0.003
19357.	18329.	520.	0.913E 13	0.0003	200.8	0.125E 13	1.	7218.	0.	0.006
19493.	18345.	514.	0.926E 13	0.0003	200.9	0.126E 13	1.	7199.	0.	0.009
19633.	18364.	508.	0.939E 13	0.0003	201.0	0.128E 13	1.	7176.	0.	0.011
19776.	18386.	502.	0.953E 13	0.0003	201.0	0.129E 13	1.	7150.	0.	0.014
19923.	18409.	496.	0.967E 13	0.0003	201.1	0.131E 13	0.	7121.	0.	0.016
20071.	18434.	490.	0.981E 13	0.0003	201.3	0.132E 13	0.	7091.	0.	0.018
20222.	18460.	485.	0.996E 13	0.0003	201.4	0.134E 13	0.	7060.	0.	0.019
20374.	18487.	479.	0.101E 14	0.0003	201.5	0.136E 13	0.	7027.	0.	0.020
20526.	18514.	474.	0.103E 14	0.0003	201.6	0.137E 13	0.	6995.	0.	0.020
20679.	18542.	468.	0.104E 14	0.0002	201.7	0.139E 13	0.	6963.	0.	0.019
20832.	18568.	463.	0.106E 14	0.0002	201.8	0.140E 13	0.	6932.	0.	0.018

2

ST	U	X	T	R	Z	EK	V	WT	YE
661.5	5.0	0.0000	201.3	20984.	18594.	458.	0.107E 14	0.0002	20
666.5	4.7	0.0000	201.1	21135.	18618.	453.	0.109E 14	0.0002	20
671.5	4.3	0.0000	200.9	21284.	18640.	448.	0.110E 14	0.0002	20
676.5	3.9	0.0000	200.7	21430.	18661.	443.	0.112E 14	0.0002	20
681.5	3.4	0.0000	200.6	21574.	18679.	438.	0.113E 14	0.0002	20
686.5	2.9	0.0000	200.5	21715.	18695.	434.	0.115E 14	0.0002	20
691.5	2.4	0.0000	200.4	21852.	18708.	429.	0.116E 14	0.0002	20
696.5	1.8	0.0000	200.3	21986.	18719.	424.	0.118E 14	0.0002	20
701.5	1.2	0.0000	200.3	22115.	18726.	420.	0.119E 14	0.0002	20
706.5	0.6	0.0000	200.2	22241.	18731.	415.	0.121E 14	0.0002	20
711.5	0.1	0.0000	200.2	22363.	18732.	411.	0.122E 14	0.0002	20
716.5	-0.5	0.0000	200.3	22481.	18731.	407.	0.123E 14	0.0002	20
721.5	-1.0	0.0000	200.3	22594.	18727.	402.	0.124E 14	0.0002	20
726.5	-1.6	0.0000	200.4	22704.	18721.	398.	0.126E 14	0.0002	20
731.5	-2.0	0.0000	200.5	22810.	18712.	394.	0.127E 14	0.0002	20
736.5	-2.5	0.0000	200.6	22912.	18701.	390.	0.128E 14	0.0002	20
741.5	-2.8	0.0000	200.8	23011.	18687.	386.	0.129E 14	0.0002	20
746.5	-3.2	0.0000	200.9	23108.	18672.	382.	0.130E 14	0.0002	20
751.5	-3.4	0.0000	201.1	23201.	18656.	379.	0.131E 14	0.0002	20
756.5	-3.6	0.0000	201.2	23293.	18638.	375.	0.132E 14	0.0002	20
761.5	-3.8	0.0000	201.4	23383.	18620.	371.	0.133E 14	0.0002	20
766.5	-3.9	0.0000	201.6	23471.	18600.	367.	0.134E 14	0.0002	20
771.5	-3.9	0.0000	201.7	23559.	18581.	364.	0.135E 14	0.0002	20
776.5	-3.8	0.0000	201.9	23646.	18562.	360.	0.136E 14	0.0002	20
781.5	-3.7	0.0000	202.1	23733.	18543.	357.	0.137E 14	0.0002	20
786.5	-3.6	0.0000	202.2	23821.	18524.	354.	0.138E 14	0.0002	20
791.5	-3.4	0.0000	202.4	23910.	18507.	350.	0.139E 14	0.0002	20
796.5	-3.1	0.0000	202.5	24000.	18491.	347.	0.140E 14	0.0002	20
801.5	-2.8	0.0000	202.6	24092.	18476.	344.	0.141E 14	0.0002	20
806.5	-2.5	0.0000	202.7	24185.	18462.	340.	0.143E 14	0.0002	20
811.5	-2.1	0.0000	202.8	24282.	18451.	337.	0.144E 14	0.0002	20
816.5	-1.7	0.0000	202.9	24380.	18441.	334.	0.145E 14	0.0002	20
821.5	-1.3	0.0000	202.9	24482.	18434.	331.	0.146E 14	0.0002	20
826.5	-0.9	0.0000	203.0	24586.	18428.	328.	0.147E 14	0.0002	20
831.5	-0.5	0.0000	203.0	24694.	18425.	325.	0.149E 14	0.0002	20
836.5	-0.0	0.0000	203.0	24804.	18423.	322.	0.150E 14	0.0002	20
841.5	0.4	0.0000	202.9	24918.	18424.	319.	0.151E 14	0.0002	20
846.5	0.8	0.0000	202.9	25035.	18427.	316.	0.153E 14	0.0002	20
851.5	1.1	0.0000	202.8	25155.	18432.	313.	0.154E 14	0.0002	20
856.5	1.5	0.0000	202.8	25278.	18438.	310.	0.156E 14	0.0002	20
861.5	1.8	0.0000	202.7	25404.	18447.	308.	0.157E 14	0.0002	20
866.5	2.1	0.0000	202.6	25532.	18456.	305.	0.159E 14	0.0001	20
871.5	2.3	0.0000	202.5	25662.	18467.	302.	0.160E 14	0.0001	20
876.5	2.5	0.0000	202.4	25795.	18480.	300.	0.162E 14	0.0001	20
881.5	2.7	0.0000	202.2	25929.	18493.	297.	0.164E 14	0.0001	20
886.5	2.8	0.0000	202.1	26064.	18506.	295.	0.165E 14	0.0001	20
891.5	2.9	0.0000	202.0	26200.	18521.	292.	0.167E 14	0.0001	20
896.5	2.9	0.0000	201.9	26337.	18535.	290.	0.169E 14	0.0001	20
901.5	2.8	0.0000	201.7	26473.	18549.	287.	0.171E 14	0.0001	20



R	Z	EK	V	WT	TE	M	ES	P	PW	ED	
1.3	20984.	18594.	458.	0.107E 14	0.0002	202.0	0.142E 13	0.	6902.	0.	0.016
1.1	21135.	18618.	453.	0.109E 14	0.0002	202.1	0.144E 13	0.	6874.	0.	0.014
0.9	21284.	18640.	448.	0.110E 14	0.0002	202.2	0.145E 13	0.	6848.	0.	0.012
0.7	21430.	18661.	443.	0.112E 14	0.0002	202.3	0.147E 13	0.	6824.	0.	0.010
0.6	21574.	18679.	438.	0.113E 14	0.0002	202.4	0.148E 13	0.	6803.	0.	0.007
0.5	21715.	18695.	434.	0.115E 14	0.0002	202.5	0.150E 13	0.	6785.	0.	0.005
0.4	21852.	18708.	429.	0.116E 14	0.0002	202.5	0.152E 13	0.	6770.	0.	0.003
0.3	21986.	18719.	424.	0.118E 14	0.0002	202.5	0.153E 13	0.	6758.	0.	0.002
0.3	22115.	18726.	420.	0.119E 14	0.0002	202.5	0.155E 13	0.	6749.	0.	0.001
0.2	22241.	18731.	415.	0.121E 14	0.0002	202.6	0.157E 13	0.	6744.	0.	0.000
0.2	22363.	18732.	411.	0.122E 14	0.0002	202.6	0.158E 13	0.	6742.	0.	0.000
0.3	22481.	18731.	407.	0.123E 14	0.0002	202.6	0.160E 13	0.	6743.	0.	0.000
0.3	22594.	18727.	402.	0.124E 14	0.0002	202.6	0.162E 13	0.	6748.	0.	0.001
0.4	22704.	18721.	398.	0.126E 14	0.0002	202.5	0.163E 13	0.	6755.	0.	0.001
0.5	22810.	18712.	394.	0.127E 14	0.0002	202.5	0.165E 13	0.	6766.	0.	0.002
0.6	22912.	18701.	390.	0.128E 14	0.0002	202.4	0.167E 13	0.	6778.	0.	0.004
0.8	23011.	18687.	386.	0.129E 14	0.0002	202.4	0.168E 13	0.	6793.	0.	0.005
0.9	23108.	18672.	382.	0.130E 14	0.0002	202.3	0.170E 13	0.	6811.	0.	0.006
1.1	23201.	18656.	379.	0.131E 14	0.0002	202.2	0.172E 13	0.	6830.	0.	0.007
1.2	23293.	18638.	375.	0.132E 14	0.0002	202.2	0.174E 13	0.	6850.	0.	0.008
1.4	23383.	18620.	371.	0.133E 14	0.0002	202.1	0.175E 13	0.	6872.	0.	0.008
1.6	23471.	18600.	367.	0.134E 14	0.0002	202.0	0.177E 13	0.	6894.	0.	0.009
1.7	23559.	18581.	364.	0.135E 14	0.0002	201.9	0.179E 13	0.	6917.	0.	0.009
1.9	23646.	18562.	360.	0.136E 14	0.0002	201.8	0.181E 13	0.	6939.	0.	0.009
2.1	23733.	18543.	357.	0.137E 14	0.0002	201.7	0.182E 13	0.	6962.	0.	0.008
2.2	23821.	18524.	354.	0.138E 14	0.0002	201.7	0.184E 13	0.	6983.	0.	0.007
2.4	23910.	18507.	350.	0.139E 14	0.0002	201.6	0.186E 13	0.	7004.	0.	0.007
2.5	24000.	18491.	347.	0.140E 14	0.0002	201.5	0.188E 13	0.	7023.	0.	0.006
2.6	24092.	18476.	344.	0.141E 14	0.0002	201.4	0.190E 13	0.	7041.	0.	0.005
2.7	24185.	18462.	340.	0.143E 14	0.0002	201.4	0.191E 13	0.	7057.	0.	0.003
2.8	24282.	18451.	337.	0.144E 14	0.0002	201.3	0.193E 13	0.	7071.	0.	0.003
2.9	24380.	18441.	334.	0.145E 14	0.0002	201.3	0.195E 13	0.	7082.	0.	0.002
2.9	24482.	18434.	331.	0.146E 14	0.0002	201.3	0.197E 13	0.	7092.	0.	0.001
3.0	24586.	18428.	328.	0.147E 14	0.0002	201.2	0.199E 13	1.	7098.	0.	0.000
3.0	24694.	18425.	325.	0.149E 14	0.0002	201.2	0.201E 13	1.	7102.	0.	0.000
3.0	24804.	18423.	322.	0.150E 14	0.0002	201.2	0.202E 13	1.	7104.	0.	0.000
2.9	24918.	18424.	319.	0.151E 14	0.0002	201.2	0.204E 13	0.	7103.	0.	0.000
2.9	25035.	18427.	316.	0.153E 14	0.0002	201.2	0.206E 13	0.	7100.	0.	0.000
2.8	25155.	18432.	313.	0.154E 14	0.0002	201.2	0.208E 13	0.	7094.	0.	0.001
2.8	25278.	18438.	310.	0.156E 14	0.0002	201.3	0.210E 13	0.	7086.	0.	0.001
2.7	25404.	18447.	308.	0.157E 14	0.0002	201.3	0.212E 13	0.	7076.	0.	0.002
2.6	25532.	18456.	305.	0.159E 14	0.0001	201.4	0.214E 13	0.	7064.	0.	0.002
2.5	25662.	18467.	302.	0.160E 14	0.0001	201.4	0.216E 13	0.	7051.	0.	0.003
2.4	25795.	18480.	300.	0.162E 14	0.0001	201.5	0.217E 13	0.	7036.	0.	0.003
2.2	25929.	18493.	297.	0.164E 14	0.0001	201.5	0.219E 13	0.	7021.	0.	0.004
2.1	26064.	18506.	295.	0.165E 14	0.0001	201.6	0.221E 13	0.	7005.	0.	0.004
2.0	26200.	18521.	292.	0.167E 14	0.0001	201.6	0.223E 13	0.	6988.	0.	0.004
1.9	26337.	18535.	290.	0.169E 14	0.0001	201.7	0.225E 13	0.	6971.	0.	0.004
1.7	26473.	18549.	287.	0.171E 14	0.0001	201.8	0.227E 13	0.	6954.	0.	0.004



UST= 0.06250 K2= 0.1250000 LAMROA= 0.250000 C1= 0.8000000 C2= 0.0000400  
 TEO=300.000000 CHANGE= 30.000 OST2= 5.0000 80= 1091.00 B1= 0.1328  
 A1= 0.00650 A2=-0.00440 A3=-0.00220 A4= 0. Z1= 16500. Z2= 22000.  
 ZT 16500. D0= 1910.0 D1= 0.03490 D2=0.00031300 TF= 273.0 C3=0.

ST	U	X	T	R	Z	EK	V	WT	TE
0.	0.	0.3329	3000.0	231.	0.	0.	0.514E 08	0.	300.0
1.0	14.9	0.3230	2962.6	231.	8.	5.	0.519E 08	0.	300.0
2.0	29.0	0.2963	2853.0	234.	30.	72.	0.534E 08	0.	299.8
3.0	41.3	0.2595	2679.9	237.	65.	301.	0.557E 08	0.	299.6
4.0	51.5	0.2199	2458.4	241.	111.	734.	0.586E 08	0.	299.3
5.0	59.4	0.1826	2208.4	246.	167.	1314.	0.620E 08	0.	298.9
6.0	64.8	0.1503	1951.5	250.	229.	1920.	0.657E 08	0.	298.5
7.0	68.2	0.1236	1705.5	255.	296.	2443.	0.698E 08	0.	298.1
8.0	69.3	0.1013	1474.2	261.	365.	2821.	0.743E 08	0.	297.6
9.0	68.6	0.0834	1268.8	267.	434.	3012.	0.794E 08	0.	297.2
10.0	66.7	0.0694	1095.6	273.	502.	3035.	0.852E 08	0.	296.7
11.0	64.3	0.0586	954.5	280.	567.	2939.	0.918E 08	0.	296.3
12.0	61.6	0.0503	841.4	287.	630.	2776.	0.991E 08	0.	295.9
13.0	58.8	0.0439	751.6	295.	690.	2583.	0.107E 09	0.	295.5
14.0	56.2	0.0389	680.1	303.	748.	2383.	0.116E 09	0.	295.1
15.0	53.7	0.0350	623.0	311.	803.	2192.	0.126E 09	0.	294.8
16.0	51.4	0.0318	577.0	320.	856.	2014.	0.137E 09	0.	294.4
17.0	49.3	0.0293	539.7	329.	906.	1853.	0.149E 09	0.	294.1
18.0	47.4	0.0272	509.1	337.	954.	1708.	0.161E 09	0.	293.8
19.0	45.6	0.0255	483.7	346.	1001.	1578.	0.174E 09	0.	293.5
20.0	44.0	0.0240	462.5	356.	1046.	1463.	0.188E 09	0.	293.2
21.0	42.5	0.0228	444.7	365.	1089.	1361.	0.203E 09	0.	292.9
22.0	41.2	0.0217	429.5	374.	1131.	1270.	0.219E 09	0.	292.7
23.0	39.9	0.0208	416.5	383.	1171.	1188.	0.235E 09	0.	292.4
24.0	38.8	0.0200	405.2	392.	1211.	1115.	0.252E 09	0.	292.1
25.0	37.7	0.0193	395.4	401.	1249.	1049.	0.269E 09	0.	291.9
26.0	36.7	0.0187	386.9	409.	1286.	990.	0.288E 09	0.	291.6
27.0	35.8	0.0181	379.3	418.	1322.	937.	0.307E 09	0.	291.4
28.0	35.0	0.0176	372.6	427.	1358.	888.	0.326E 09	0.	291.2
29.0	34.2	0.0171	366.7	436.	1392.	844.	0.346E 09	0.	290.9
30.0	33.5	0.0167	361.3	444.	1426.	803.	0.367E 09	0.	290.7
35.0	30.4	0.0151	341.4	485.	1585.	646.	0.479E 09	0.	289.7
40.0	28.0	0.0140	328.5	525.	1731.	537.	0.605E 09	0.	288.7
45.0	26.2	0.0131	319.5	562.	1867.	459.	0.743E 09	0.	287.9
50.0	24.7	0.0124	313.0	597.	1994.	400.	0.892E 09	0.	287.0
55.0	23.4	0.0118	307.9	631.	2114.	355.	0.105E 10	0.	286.3
60.0	22.3	0.0114	303.9	663.	2228.	318.	0.122E 10	0.	285.5
65.0	21.3	0.0109	300.6	695.	2337.	288.	0.140E 10	0.	284.8
70.0	20.5	0.0106	297.8	725.	2441.	264.	0.159E 10	0.	284.1
75.0	19.7	0.0102	295.5	754.	2542.	243.	0.179E 10	0.	283.5
80.0	19.0	0.0099	293.4	782.	2638.	225.	0.200E 10	0.	282.9
85.0	18.3	0.0096	291.6	809.	2732.	210.	0.222E 10	0.	282.2
90.0	17.8	0.0094	290.0	836.	2822.	196.	0.245E 10	0.	281.7
95.0	17.2	0.0091	288.6	862.	2909.	184.	0.268E 10	0.	281.1
100.0	16.7	0.0089	287.2	887.	2994.	174.	0.292E 10	0.	280.5
105.0	16.2	0.0087	286.0	912.	3076.	164.	0.317E 10	0.	280.0
110.0	15.7	0.0085	284.9	936.	3156.	156.	0.343E 10	0.	279.5



LAMROA= 0.250000 C1= 0.8000000 C2= 0.0000400 W=0.20000E 02 F= 0.33333333 PHI= 0.50000000  
 .000 DST2= 5.0000 80- 1091.00 81= 0.1328 82= 0. DST1= 1.0 K=2  
 A3=-0.00220 A4= 0. Z1= 16500. Z2= 22000. Z3= 52000. P0= 101300.  
 D1= 0.03490 D2=0.00031300 TF= 273.0 C3=0. PRINT= 400.0 RK3= 1.0

R	Z	EK	V	WT	TE	M	ES	P	PW	ED	
.0	231.	0.	0.	0.514E 08	0.	300.0	0.525E 07	20666449.	101300.	35362.	0.004
.6	231.	8.	5.	0.519E 08	0.	300.0	0.538E 07	21417101.	101214.	34643.	20.171
.0	234.	30.	72.	0.534E 08	0.	299.8	0.578E 07	23784952.	100961.	32621.	140.432
.9	237.	65.	301.	0.557E 08	0.	299.6	0.646E 07	28094935.	100556.	29646.	374.041
.4	241.	111.	734.	0.586E 08	0.	299.3	0.748E 07	34769876.	100024.	26165.	645.724
.4	246.	167.	1314.	0.620E 08	0.	298.9	0.888E 07	44018838.	99390.	22596.	858.308
.5	250.	229.	1920.	0.657E 08	0.	298.5	0.107E 08	55265278.	98685.	19241.	957.731
.5	255.	296.	2443.	0.698E 08	0.	298.1	0.131E 08	66613828.	97934.	16263.	966.799
.2	261.	365.	2821.	0.743E 08	0.	297.6	0.162E 08	75093236.	97164.	13635.	897.704
.8	267.	434.	3012.	0.794E 08	0.	297.2	0.201E 08	76629784.	96396.	11416.	760.106
.6	273.	502.	3035.	0.852E 08	0.	296.7	0.250E 08	69518774.	95648.	9614.	606.107
.5	280.	567.	2939.	0.918E 08	0.	296.3	0.308E 08	56142861.	94928.	8185.	468.316
.4	287.	630.	2776.	0.991E 08	0.	295.9	0.376E 08	41016207.	94241.	7061.	357.671
.6	295.	690.	2583.	0.107E 09	0.	295.5	0.454E 08	27758402.	93588.	6179.	273.427
.1	303.	748.	2383.	0.116E 09	0.	295.1	0.542E 08	17848132.	92968.	5481.	210.763
.0	311.	803.	2192.	0.126E 09	0.	294.8	0.640E 08	11155743.	92378.	4925.	164.446
.0	320.	856.	2014.	0.137E 09	0.	294.4	0.746E 08	6905892.	91817.	4477.	130.098
.7	329.	906.	1853.	0.149E 09	0.	294.1	0.861E 08	4293851.	91281.	4111.	104.407
.1	337.	954.	1708.	0.161E 09	0.	293.8	0.985E 08	2707887.	90770.	3809.	84.970
.7	346.	1001.	1578.	0.174E 09	0.	293.5	0.112E 09	1743040.	90281.	3557.	70.076
.5	356.	1046.	1463.	0.188E 09	0.	293.2	0.126E 09	1149351.	89811.	3344.	58.514
.7	365.	1089.	1361.	0.203E 09	0.	292.9	0.140E 09	777676.	89360.	3162.	49.422
.5	374.	1131.	1270.	0.219E 09	0.	292.7	0.156E 09	540123.	88925.	3005.	42.182
.5	383.	1171.	1138.	0.235E 09	0.	292.4	0.172E 09	384853.	88505.	2869.	36.350
.2	392.	1211.	1115.	0.252E 09	0.	292.1	0.189E 09	281012.	88099.	2749.	31.598
.4	401.	1249.	1049.	0.269E 09	0.	291.9	0.206E 09	209975.	87706.	2643.	27.687
.9	409.	1286.	990.	0.288E 09	0.	291.6	0.224E 09	160301.	87325.	2548.	24.436
.3	418.	1322.	937.	0.307E 09	0.	291.4	0.243E 09	124833.	86954.	2464.	21.711
.6	427.	1358.	888.	0.326E 09	0.	291.2	0.262E 09	99004.	86594.	2387.	19.406
.7	436.	1392.	844.	0.346E 09	0.	290.9	0.281E 09	79845.	86243.	2317.	17.442
.3	444.	1426.	803.	0.367E 09	0.	290.7	0.301E 09	65387.	85901.	2254.	15.757
.4	485.	1585.	646.	0.479E 09	0.	289.7	0.409E 09	29029.	84305.	2003.	10.164
.5	525.	1731.	537.	0.605E 09	0.	288.7	0.528E 09	16147.	82866.	1823.	7.033
.5	562.	1867.	459.	0.743E 09	0.	287.9	0.656E 09	10395.	81548.	1686.	5.157
.0	597.	1994.	400.	0.892E 09	0.	287.0	0.793E 09	7381.	80327.	1575.	3.943
.9	631.	2114.	355.	0.105E 10	0.	286.3	0.938E 09	5609.	79186.	1483.	3.113
.9	663.	2228.	318.	0.122E 10	0.	285.5	0.109E 10	4475.	78115.	1404.	2.518
.6	695.	2337.	288.	0.140E 10	0.	284.8	0.125E 10	3701.	77102.	1335.	2.078
.8	725.	2441.	264.	0.159E 10	0.	284.1	0.141E 10	3144.	76142.	1274.	1.742
.5	754.	2542.	243.	0.179E 10	0.	283.5	0.158E 10	2728.	75228.	1219.	1.480
.4	782.	2638.	225.	0.200E 10	0.	282.9	0.176E 10	2406.	74356.	1170.	1.271
.6	809.	2732.	210.	0.222E 10	0.	282.2	0.194E 10	2150.	73522.	1124.	1.102
0.0	836.	2822.	196.	0.245E 10	0.	281.7	0.213E 10	1943.	72723.	1083.	0.963
.6	862.	2909.	184.	0.268E 10	0.	281.1	0.232E 10	1771.	71956.	1044.	0.847
.2	887.	2994.	174.	0.292E 10	0.	280.5	0.252E 10	1626.	71219.	1008.	0.750
.0	912.	3076.	164.	0.317E 10	0.	280.0	0.272E 10	1503.	70510.	975.	0.667
.9	936.	3156.	156.	0.343E 10	0.	279.5	0.292E 10	1396.	69826.	944.	0.59

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ST	U	X	T	R	Z	EK	V	WT	TE
115.0	15.3	0.0083	283.9	959.	3233.	148.	0.370E 10	0.	279.0
120.0	14.9	0.0081	282.9	982.	3309.	141.	0.397E 10	0.	278.5
125.0	14.5	0.0080	282.0	1005.	3382.	135.	0.425E 10	0.	278.0
130.0	14.1	0.0078	281.2	1027.	3453.	129.	0.454E 10	0.	277.6
135.0	13.7	0.0077	280.4	1049.	3523.	124.	0.484E 10	0.	277.1
140.0	13.3	0.0075	279.6	1070.	3590.	119.	0.514E 10	0.	276.7
145.0	13.0	0.0074	278.9	1092.	3656.	114.	0.545E 10	0.	276.2
150.0	12.6	0.0072	278.2	1112.	3720.	110.	0.576E 10	0.	275.8
155.0	12.3	0.0071	277.5	1132.	3782.	106.	0.608E 10	0.	275.4
160.0	11.9	0.0070	276.9	1152.	3842.	102.	0.641E 10	0.	275.0
165.0	11.6	0.0069	276.3	1172.	3901.	98.	0.675E 10	0.	274.6
170.0	11.3	0.0068	275.8	1191.	3959.	95.	0.709E 10	0.	274.3
175.0	11.0	0.0067	275.2	1211.	4014.	92.	0.743E 10	0.	273.9
180.0	10.7	0.0066	274.7	1229.	4069.	89.	0.778E 10	0.	273.6
185.0	10.4	0.0065	274.2	1248.	4121.	86.	0.814E 10	0.	273.2
190.0	10.1	0.0064	273.7	1266.	4173.	84.	0.850E 10	0.	272.9
195.0	9.8	0.0063	273.2	1284.	4222.	81.	0.887E 10	0.	272.6
200.0	9.5	0.0062	272.8	1302.	4271.	79.	0.924E 10	0.	272.2
205.0	9.2	0.0061	272.4	1319.	4318.	77.	0.962E 10	0.	271.9
200.5	9.5	0.0062	272.8	1303.	4275.	79.	0.928E 10	0.	272.2
201.0	9.5	0.0062	272.7	1305.	4280.	78.	0.931E 10	0.	272.2
201.5	9.4	0.0061	272.7	1307.	4285.	78.	0.935E 10	0.	272.1
202.0	9.4	0.0061	272.6	1309.	4290.	78.	0.939E 10	0.	272.1
202.5	9.4	0.0061	272.6	1310.	4294.	78.	0.943E 10	0.	272.1
203.0	9.3	0.0061	272.5	1312.	4299.	78.	0.946E 10	0.	272.1
SWITCH TO WET									
208.0	9.1	0.0060	272.2	1328.	4345.	75.	0.982E 10	0.0000	271.8
213.0	8.8	0.0059	271.9	1345.	4390.	73.	0.102E 11	0.0001	271.5
218.0	8.6	0.0058	271.6	1360.	4433.	71.	0.105E 11	0.0001	271.2
223.0	8.3	0.0056	271.3	1376.	4475.	69.	0.109E 11	0.0001	270.9
228.0	8.1	0.0055	271.0	1392.	4517.	68.	0.113E 11	0.0002	270.6
233.0	7.9	0.0054	270.8	1407.	4557.	66.	0.117E 11	0.0002	270.4
238.0	7.7	0.0053	270.5	1422.	4596.	64.	0.120E 11	0.0002	270.1
243.0	7.5	0.0053	270.2	1437.	4634.	63.	0.124E 11	0.0002	269.9
248.0	7.4	0.0052	270.0	1452.	4671.	61.	0.128E 11	0.0003	269.6
253.0	7.2	0.0051	269.7	1466.	4708.	60.	0.132E 11	0.0003	269.4
258.0	7.0	0.0050	269.5	1481.	4743.	58.	0.136E 11	0.0003	269.2
263.0	6.9	0.0049	269.2	1495.	4778.	57.	0.140E 11	0.0003	268.9
268.0	6.7	0.0048	269.0	1509.	4812.	56.	0.144E 11	0.0004	268.7
273.0	6.6	0.0048	268.7	1523.	4845.	54.	0.148E 11	0.0004	268.5
278.0	6.4	0.0047	268.5	1537.	4878.	53.	0.152E 11	0.0004	268.3
283.0	6.3	0.0046	268.3	1550.	4909.	52.	0.156E 11	0.0004	268.1
288.0	6.2	0.0046	268.1	1564.	4941.	51.	0.160E 11	0.0004	267.9
293.0	6.0	0.0045	267.9	1577.	4971.	50.	0.164E 11	0.0004	267.7
298.0	5.9	0.0044	267.7	1590.	5001.	49.	0.168E 11	0.0004	267.5
303.0	5.8	0.0044	267.5	1503.	5030.	48.	0.173E 11	0.0004	267.3
308.0	5.7	0.0043	267.3	1616.	5059.	47.	0.177E 11	0.0005	267.1
313.0	5.6	0.0042	267.1	1629.	5087.	46.	0.181E 11	0.0005	266.9
318.0	5.4	0.0042	266.9	1642.	5114.	45.	0.185E 11	0.0005	266.8
323.0	5.3	0.0041	266.7	1654.	5141.	44.	0.190E 11	0.0005	266.6
328.0	5.2	0.0041	266.5	1667.	5168.	43.	0.194E 11	0.0005	266.4
333.0	5.1	0.0040	266.3	1679.	5193.	43.	0.198E 11	0.0005	266.2
338.0	5.0	0.0040	266.1	1691.	5219.	42.	0.203E 11	0.0005	266.1
343.0	4.9	0.0039	266.0	1703.	5244.	41.	0.207E 11	0.0005	265.9
348.0	4.8	0.0039	265.8	1715.	5268.	40.	0.211E 11	0.0005	265.8
353.0	4.7	0.0038	265.6	1727.	5292.	40.	0.216E 11	0.0005	265.6
358.0	4.6	0.0038	265.5	1739.	5315.	39.	0.220E 11	0.0005	265.5



R	Z	EK	V	WT	TE	M	ES	P	PW	EC
.9	959.	3233.	148.	0.370E 10 0.	279.0	0.313E 10	1303.	69168.	915.	0.534
.9	982.	3309.	141.	0.397E 10 0.	278.5	0.334E 10	1222.	68532.	887.	0.480
0	1005.	3382.	135.	0.425E 10 0.	278.0	0.356E 10	1150.	67919.	862.	0.433
.2	1027.	3453.	129.	0.454E 10 0.	277.6	0.378E 10	1085.	67326.	837.	0.392
.4	1049.	3523.	124.	0.484E 10 0.	277.1	0.400E 10	1027.	66754.	814.	0.355
.6	1070.	3590.	119.	0.514E 10 0.	276.7	0.423E 10	975.	66201.	792.	0.323
.9	1092.	3656.	114.	0.545E 10 0.	276.2	0.446E 10	928.	65666.	771.	0.294
.2	1112.	3720.	110.	0.576E 10 0.	275.8	0.469E 10	884.	65148.	752.	0.268
.5	1132.	3782.	106.	0.608E 10 0.	275.4	0.493E 10	845.	64648.	733.	0.244
.9	1152.	3842.	102.	0.641E 10 0.	275.0	0.517E 10	809.	64164.	715.	0.223
.3	1172.	3901.	98.	0.675E 10 0.	274.6	0.541E 10	775.	63696.	698.	0.204
.8	1191.	3959.	95.	0.704E 10 0.	274.3	0.565E 10	745.	63244.	682.	0.186
.2	1211.	4014.	92.	0.743E 10 0.	273.9	0.590E 10	716.	62806.	666.	0.170
.7	1229.	4069.	89.	0.778E 10 0.	273.6	0.615E 10	690.	62383.	652.	0.156
.2	1248.	4121.	86.	0.814E 10 0.	273.2	0.640E 10	666.	61973.	637.	0.143
.7	1266.	4173.	84.	0.850E 10 0.	272.9	0.666E 10	643.	61578.	624.	0.131
.2	1284.	4222.	81.	0.887E 10 0.	272.6	0.691E 10	622.	61195.	611.	0.120
.8	1302.	4271.	79.	0.924E 10 0.	272.2	0.717E 10	602.	60826.	599.	0.110
.4	1319.	4318.	77.	0.962E 10 0.	271.9	0.743E 10	584.	60469.	587.	0.100
.8	1303.	4275.	79.	0.928E 10 0.	272.2	0.720E 10	600.	60790.	597.	0.109
.7	1305.	4280.	78.	0.931E 10 0.	272.2	0.722E 10	598.	60754.	596.	0.108
.7	1307.	4285.	78.	0.935E 10 0.	272.1	0.725E 10	597.	60718.	595.	0.107
.6	1309.	4290.	78.	0.939E 10 0.	272.1	0.727E 10	595.	60682.	594.	0.106
.6	1310.	4294.	78.	0.943E 10 0.	272.1	0.730E 10	593.	60646.	593.	0.105
.5	1312.	4299.	78.	0.946E 10 0.	272.1	0.733E 10	591.	60610.	591.	0.104
.2	1328.	4345.	75.	0.982E 10 0.0000	271.8	0.759E 10	577.	60261.	576.	0.095
.9	1345.	4390.	73.	0.102E 11 0.0001	271.5	0.785E 10	564.	59923.	561.	0.088
.6	1360.	4433.	71.	0.105E 11 0.0001	271.2	0.812E 10	552.	59596.	548.	0.081
.3	1376.	4475.	69.	0.109E 11 0.0001	270.9	0.839E 10	540.	59279.	534.	0.075
.0	1392.	4517.	68.	0.113E 11 0.0002	270.6	0.866E 10	529.	58972.	522.	0.069
.8	1407.	4557.	66.	0.117E 11 0.0002	270.4	0.893E 10	518.	58674.	510.	0.064
.5	1422.	4596.	64.	0.120E 11 0.0002	270.1	0.920E 10	508.	58385.	498.	0.060
.2	1437.	4634.	63.	0.124E 11 0.0002	269.9	0.948E 10	498.	58104.	487.	0.055
.0	1452.	4671.	61.	0.128E 11 0.0003	269.6	0.976E 10	488.	57831.	477.	0.052
.7	1466.	4708.	60.	0.132E 11 0.0003	269.4	0.100E 11	479.	57565.	467.	0.048
.5	1481.	4743.	58.	0.136E 11 0.0003	269.2	0.103E 11	470.	57306.	457.	0.045
.2	1495.	4778.	57.	0.140E 11 0.0003	268.9	0.106E 11	462.	57054.	448.	0.042
.0	1509.	4812.	56.	0.144E 11 0.0004	268.7	0.109E 11	454.	56808.	439.	0.039
.7	1523.	4845.	54.	0.148E 11 0.0004	268.5	0.112E 11	446.	56569.	431.	0.037
.5	1537.	4878.	53.	0.152E 11 0.0004	268.3	0.115E 11	438.	56335.	422.	0.035
.3	1550.	4909.	52.	0.156E 11 0.0004	268.1	0.117E 11	431.	56108.	414.	0.033
.1	1564.	4941.	51.	0.160E 11 0.0004	257.9	0.120E 11	424.	55885.	407.	0.031
.9	1577.	4971.	50.	0.164E 11 0.0004	267.7	0.123E 11	417.	55669.	400.	0.029
.7	1590.	5001.	49.	0.168E 11 0.0004	267.5	0.126E 11	411.	55457.	393.	0.027
.5	1603.	5030.	48.	0.173E 11 0.0004	267.3	0.129E 11	404.	55250.	306.	0.026
.3	1616.	5059.	47.	0.177E 11 0.0005	267.1	0.132E 11	398.	55048.	379.	0.024
.1	1629.	5087.	46.	0.181E 11 0.0005	266.9	0.135E 11	392.	54851.	373.	0.023
.9	1642.	5114.	45.	0.185E 11 0.0005	266.8	0.138E 11	387.	54659.	367.	0.021
.7	1654.	5141.	44.	0.190E 11 0.0005	266.6	0.141E 11	381.	54471.	361.	0.020
.5	1667.	5168.	43.	0.194E 11 0.0005	266.4	0.144E 11	376.	54287.	355.	0.019
.3	1679.	5193.	43.	0.198E 11 0.0005	266.2	0.147E 11	370.	54108.	350.	0.018
.1	1691.	5219.	42.	0.203E 11 0.0005	266.1	0.150E 11	365.	53932.	344.	0.017
.0	1703.	5244.	41.	0.207E 11 0.0005	265.9	0.153E 11	361.	53761.	339.	0.016
.8	1715.	5268.	40.	0.211E 11 0.0005	265.8	0.156E 11	356.	53594.	334.	0.015
.6	1727.	5292.	40.	0.216E 11 0.0005	265.6	0.159E 11	351.	53431.	329.	0.014
.5	1739.	5315.	39.	0.220E 11 0.0005	265.5	0.162E 11	347.	53271.	325.	0.013

2

ST	U	X	T	R	Z	EK	V	WT	TE
363.0	4.5	0.0038	265.3	1751.	5338.	38.	0.225E 11	0.0005	265.
368.0	4.4	0.0037	265.1	1762.	5360.	38.	0.229E 11	0.0005	265.
373.0	4.3	0.0037	265.0	1774.	5382.	37.	0.234E 11	0.0005	265.
378.0	4.2	0.0036	264.8	1785.	5403.	36.	0.238E 11	0.0006	264.
383.0	4.1	0.0036	264.7	1797.	5424.	36.	0.243E 11	0.0006	264.
388.0	4.0	0.0036	264.5	1808.	5444.	35.	0.248E 11	0.0006	264.
393.0	3.9	0.0035	264.4	1819.	5464.	35.	0.252E 11	0.0006	264.
398.0	3.8	0.0035	264.3	1830.	5483.	34.	0.257E 11	0.0006	264.
403.0	3.7	0.0035	264.1	1841.	5502.	34.	0.261E 11	0.0006	264.
408.0	3.7	0.0034	264.0	1852.	5521.	33.	0.266E 11	0.0006	264.
413.0	3.6	0.0034	263.9	1863.	5539.	33.	0.271E 11	0.0006	264.
418.0	3.5	0.0034	263.7	1873.	5557.	32.	0.275E 11	0.0006	263.
423.0	3.4	0.0033	263.6	1884.	5574.	32.	0.280E 11	0.0006	263.
428.0	3.3	0.0033	263.5	1894.	5590.	31.	0.285E 11	0.0006	263.
433.0	3.2	0.0033	263.4	1905.	5607.	31.	0.289E 11	0.0006	263.
438.0	3.1	0.0033	263.3	1915.	5622.	30.	0.294E 11	0.0006	263.
443.0	3.0	0.0032	263.2	1925.	5638.	30.	0.299E 11	0.0006	263.
448.0	2.9	0.0032	263.0	1935.	5653.	29.	0.304E 11	0.0006	263.
453.0	2.8	0.0032	262.9	1946.	5667.	29.	0.308E 11	0.0006	263.
458.0	2.8	0.0032	262.8	1956.	5681.	29.	0.313E 11	0.0006	263.
463.0	2.7	0.0031	262.7	1965.	5695.	28.	0.318E 11	0.0006	263.
468.0	2.6	0.0031	262.6	1975.	5708.	28.	0.323E 11	0.0006	262.
473.0	2.5	0.0031	262.5	1985.	5720.	27.	0.328E 11	0.0006	262.
478.0	2.4	0.0031	262.5	1995.	5733.	27.	0.333E 11	0.0005	262.
483.0	2.3	0.0031	262.4	2004.	5744.	27.	0.337E 11	0.0005	262.
488.0	2.2	0.0030	262.3	2014.	5756.	26.	0.342E 11	0.0005	262.
493.0	2.1	0.0030	262.2	2023.	5766.	26.	0.347E 11	0.0005	262.
498.0	2.0	0.0030	262.1	2033.	5777.	26.	0.352E 11	0.0005	262.
503.0	1.9	0.0030	262.0	2042.	5787.	25.	0.357E 11	0.0005	262.
508.0	1.9	0.0030	262.0	2051.	5796.	25.	0.362E 11	0.0005	262.
513.0	1.8	0.0030	261.9	2061.	5805.	25.	0.366E 11	0.0005	262.
518.0	1.7	0.0029	261.8	2070.	5814.	24.	0.371E 11	0.0005	262.
523.0	1.6	0.0029	261.8	2079.	5822.	24.	0.376E 11	0.0005	262.
528.0	1.5	0.0029	261.7	2088.	5830.	24.	0.381E 11	0.0005	262.
533.0	1.4	0.0029	261.7	2097.	5837.	24.	0.386E 11	0.0005	262.
538.0	1.3	0.0029	261.6	2105.	5844.	23.	0.391E 11	0.0005	262.
543.0	1.2	0.0029	261.5	2114.	5850.	23.	0.396E 11	0.0005	262.
548.0	1.1	0.0029	261.5	2123.	5856.	23.	0.401E 11	0.0005	261.
553.0	1.0	0.0029	261.4	2131.	5861.	22.	0.406E 11	0.0005	261.
558.0	0.9	0.0029	261.4	2140.	5866.	22.	0.411E 11	0.0005	261.
563.0	0.8	0.0028	261.4	2148.	5870.	22.	0.415E 11	0.0005	261.
568.0	0.7	0.0028	261.3	2157.	5874.	22.	0.420E 11	0.0004	261.
573.0	0.6	0.0028	261.3	2165.	5878.	21.	0.425E 11	0.0004	261.
578.0	0.5	0.0028	261.3	2173.	5881.	21.	0.430E 11	0.0004	261.
583.0	0.5	0.0028	261.2	2182.	5883.	21.	0.435E 11	0.0004	261.
588.0	0.4	0.0028	261.2	2190.	5885.	21.	0.440E 11	0.0004	261.
593.0	0.3	0.0028	261.2	2198.	5887.	21.	0.445E 11	0.0004	261.
598.0	0.2	0.0028	261.2	2206.	5888.	20.	0.450E 11	0.0004	261.
603.0	0.1	0.0028	261.2	2214.	5888.	20.	0.454E 11	0.0004	261.



R	Z	EK	V	WT	TE	M	ES	P	PW	EC	
3	1751.	5338.	38.	0.225E 11	0.0005	265.3	0.166E 11	343.	53115.	320.	0.013
1	1762.	5360.	38.	0.229E 11	0.0005	265.2	0.169E 11	338.	52963.	316.	0.012
0	1774.	5382.	37.	0.234E 11	0.0005	265.0	0.172E 11	334.	52815.	312.	0.011
8	1785.	5403.	36.	0.238E 11	0.0006	264.9	0.175E 11	330.	52670.	308.	0.011
7	1797.	5424.	36.	0.243E 11	0.0006	264.7	0.178E 11	327.	52529.	304.	0.010
5	1808.	5444.	35.	0.248E 11	0.0006	264.6	0.181E 11	323.	52391.	300.	0.009
4	1819.	5464.	35.	0.252E 11	0.0006	264.5	0.184E 11	319.	52256.	296.	0.009
3	1830.	5483.	34.	0.257E 11	0.0006	264.4	0.188E 11	316.	52125.	293.	0.008
1	1841.	5502.	34.	0.261E 11	0.0006	264.2	0.191E 11	313.	51998.	289.	0.008
0	1852.	5521.	33.	0.266E 11	0.0006	264.1	0.194E 11	309.	51874.	286.	0.007
9	1863.	5539.	33.	0.271E 11	0.0006	264.0	0.197E 11	306.	51753.	283.	0.007
7	1873.	5557.	32.	0.275E 11	0.0006	263.9	0.200E 11	303.	51635.	279.	0.006
6	1884.	5574.	32.	0.280E 11	0.0006	263.8	0.204E 11	300.	51520.	276.	0.006
5	1894.	5590.	31.	0.285E 11	0.0006	263.7	0.207E 11	297.	51409.	273.	0.006
4	1905.	5607.	31.	0.289E 11	0.0006	263.6	0.210E 11	295.	51301.	271.	0.005
3	1915.	5622.	30.	0.294E 11	0.0006	263.5	0.213E 11	292.	51196.	268.	0.005
2	1925.	5638.	30.	0.299E 11	0.0006	263.4	0.217E 11	290.	51095.	265.	0.005
0	1935.	5653.	29.	0.304E 11	0.0006	263.3	0.220E 11	287.	50996.	263.	0.004
9	1946.	5667.	29.	0.308E 11	0.0006	263.2	0.223E 11	285.	50901.	260.	0.004
8	1956.	5681.	29.	0.313E 11	0.0006	263.1	0.226E 11	282.	50808.	258.	0.004
7	1965.	5695.	28.	0.318E 11	0.0006	263.0	0.230E 11	280.	50719.	256.	0.003
6	1975.	5708.	28.	0.323E 11	0.0006	262.9	0.233E 11	278.	50633.	254.	0.003
5	1985.	5720.	27.	0.328E 11	0.0006	262.8	0.236E 11	276.	50550.	251.	0.003
5	1995.	5733.	27.	0.333E 11	0.0005	262.7	0.240E 11	274.	50470.	249.	0.003
4	2004.	5744.	27.	0.337E 11	0.0005	262.7	0.243E 11	272.	50393.	247.	0.002
3	2014.	5756.	26.	0.342E 11	0.0005	262.6	0.246E 11	270.	50319.	246.	0.002
2	2023.	5766.	26.	0.347E 11	0.0005	262.5	0.250E 11	268.	50248.	244.	0.002
1	2033.	5777.	26.	0.352E 11	0.0005	262.5	0.253E 11	267.	50180.	242.	0.002
0	2042.	5787.	25.	0.357E 11	0.0005	262.4	0.256E 11	265.	50115.	240.	0.002
0	2051.	5796.	25.	0.362E 11	0.0005	262.3	0.260E 11	264.	50053.	239.	0.001
9	2061.	5805.	25.	0.366E 11	0.0005	262.3	0.263E 11	262.	49994.	237.	0.001
8	2070.	5814.	24.	0.371E 11	0.0005	262.2	0.266E 11	261.	49939.	236.	0.001
8	2079.	5822.	24.	0.376E 11	0.0005	262.2	0.270E 11	259.	49886.	235.	0.001
7	2088.	5830.	24.	0.381E 11	0.0005	262.1	0.273E 11	258.	49836.	233.	0.001
7	2097.	5837.	24.	0.386E 11	0.0005	262.1	0.276E 11	257.	49789.	232.	0.001
6	2105.	5844.	23.	0.391E 11	0.0005	262.0	0.280E 11	256.	49746.	231.	0.001
5	2114.	5850.	23.	0.396E 11	0.0005	262.0	0.283E 11	255.	49705.	230.	0.001
5	2123.	5856.	23.	0.401E 11	0.0005	261.9	0.287E 11	254.	49668.	229.	0.000
4	2131.	5861.	22.	0.406E 11	0.0005	261.9	0.290E 11	253.	49633.	228.	0.000
4	2140.	5866.	22.	0.411E 11	0.0005	261.9	0.293E 11	252.	49602.	227.	0.000
4	2148.	5870.	22.	0.415E 11	0.0005	261.8	0.297E 11	251.	49573.	226.	0.000
3	2157.	5874.	22.	0.420E 11	0.0004	261.8	0.300E 11	250.	49548.	225.	0.000
3	2165.	5878.	21.	0.425E 11	0.0004	261.8	0.304E 11	250.	49525.	225.	0.000
3	2173.	5881.	21.	0.430E 11	0.0004	261.8	0.307E 11	249.	49506.	224.	0.000
2	2182.	5883.	21.	0.435E 11	0.0004	261.8	0.311E 11	248.	49490.	224.	0.000
2	2190.	5885.	21.	0.440E 11	0.0004	261.7	0.314E 11	248.	49477.	223.	0.000
2	2198.	5887.	21.	0.445E 11	0.0004	261.7	0.318E 11	248.	49467.	223.	0.000
2	2206.	5888.	20.	0.450E 11	0.0004	261.7	0.321E 11	247.	49460.	222.	0.000
2	2214.	5888.	20.	0.454E 11	0.0004	261.7	0.325E 11	247.	49457.	222.	0.000



Then the cloud model would have only one empirical parameter,  $\lambda$ , whose magnitude is found to be that given in earlier studies.<sup>4, 18</sup>

Preliminary testing of the effect of atmospheric conditions on cloud rise indicates that for low yields, the initial water content of the cloud is unimportant compared with atmospheric humidity. For very high (megaton) yields the opposite is true, but the water content has only minor effect.

## SECTION 4

### OUTLINE OF FUTURE WORK ON THE WATER-SURFACE-BURST FALLOUT MODEL

#### 4.1 THE NUCLEAR CLOUD

As suggested in Sec. 2.6.4, a compressibility correction for the rise of very high yield clouds may be introduced.

The effect of wind shear on cloud rise may be considered as follows: As the ratio of the wind shear between top and bottom of cloud to the rate of rise increases, the cloud is stretched, increasing the surface-to-volume ratio and therefore the entrainment rate, and damping the cloud motion.

The cloud-rise characteristics predicted by the equations using different parameter values, will be compared with nuclear cloud observations.

#### 4.2 PARTICLE FORMATION

The atomic cloud from a sea-water-surface burst contains a large amount of water, a much smaller amount of sea salt, and a very small amount of metallic material.

As the cloud cools, the metal, salt and water condense successively. The condensed metal particles are the nuclei for particle growth. The metal-salt-water particles are called slurry particles or slurry droplets.<sup>11</sup>

We will assume that radioactivity is distributed uniformly per unit sea salt mass. Given an estimate of metal and salt masses per unit cloud volume, the initial particle masses can be taken arbitrarily small as the rate of coagulation by Brownian movement is sensitive to the mass density, not number density for small particles. (That is, it does not

matter if we suppose the mass to consist of one particle of diameter  $10^{-6}$  cm or a thousand particles of diameter  $10^{-7}$  cm per unit volume, because of the rapid growth from  $10^{-7}$  to  $10^{-6}$  cm).

As the particles reach about 1 micron ( $10^{-4}$  cm) diameter the coagulative effect of Brownian movement decreases and coagulation by turbulence<sup>19</sup> must influence further particle growth. Part of this growth occurs during the dry stage of cloud rise and part during the wet (including frozen) stage.

The temperature, liquid and solid mass per unit cloud volume, and intensity of turbulence all vary with time. Hence, approximations will be needed in analyzing the coagulation process, in order to use existing steady-state theories.

It may be that large slurry particles are formed only from megaton water-surface-bursts. This follows if the rate of generation of turbulence is proportional to the cube of rate of rise, as in Eq. (3.6), Sec. 3.6. This suggests that low-yield water-surface bursts would produce relatively little fallout, since the requisite coagulative influence would be lacking.

#### 4.3 FALLOUT OF SLURRY PARTICLES

Until now<sup>10, 11</sup> slurry-droplet mass-altitude-time histories have been calculated with the particles starting their fall either dry or wet (at sea water concentration) from a reasonable stabilized cloud height (say 60,000 ft for a megaton burst) and from various lesser altitudes.

When calculations for "initially wet" (i.e. at sea-water salt concentration) particles were compared with weapon test data<sup>10, 11</sup> it was found that particles with large salt mass came down too soon and those with small salt mass came down too late, with some crossover point at which calculated and observed times of arrival agreed.

A tentative solution to the discrepancy is as follows: in the earlier part of cloud rise, particles are being formed dry. As they

grow, some of them escape from the cloud by diffusion or gravity. The largest particles have the greatest probability of escape. Since they escape dry, they are initially lighter, and reach the ground later than wet particles with the same salt mass. The remaining "salt solution" in the cloud is diluted by the loss of salt to below sea-water salt concentration. (Entrained atmospheric moisture also contributes to this effect.) Thus, the smaller particles which stayed in the cloud longer come out wetter. Then they are larger and heavier, per unit salt mass, and reach the ground earlier than the "initially wet" particles of Refs. 10 and 11, and in better agreement with weapon test data. Farlow<sup>10</sup> has offered a similar argument.

## SECTION 5

### RESULTS AND CONCLUSIONS

#### 5.1 RESULTS

A model of the rise and expansion of the atomic cloud from a water-surface burst has been developed. A set of differential equations describes the changes with time in cloud size, shape, altitude, temperature, water content and turbulent energy density as functions of explosion energy, initial air-water energy partition and atmospheric conditions. The model is applicable to air bursts as well as water surface bursts. It uses turbulence concepts which account for the late, horizontal expansion of megaton clouds, and can also be applied to the formation of fallout particles. The equations have been programmed for machine computation. Numerical results indicate that with appropriate choice of parameters, the cloud model agrees with observations of nuclear explosion clouds. This agreement will be the subject of a separate (Classified) report.

#### 5.2 CONCLUSIONS

The cloud model gives an adequate phenomenological description of the rise and expansion of a nuclear cloud. This description can provide thermodynamic input data for development of the second part of the water-surface burst fallout model: particle formation. It can also specify the initial altitude of particles for Part 3 of the model: particle fall through the atmosphere. Part 1 of the three-part development of the fallout model is therefore accomplished. Since Part 3 of the model has already been developed, only Part 2 remains to be developed, followed by combination of the three parts into the final model.

## APPENDIX A

### SYMBOLS USED IN THE REPORT

#### A.1 A NOTE ON NOTATION

This report uses hydrodynamics, thermodynamics and meteorology. These fields use the same symbols for different quantities; consequently, any notation used must violate some usage. For example, in meteorology  $x$  and  $w$  are used for ratios of vapor- and liquid-water mass to dry air mass, respectively. But in hydrodynamics the velocity components  $u$ ,  $v$ ,  $w$  correspond to the coordinates  $x$ ,  $y$ ,  $z$ . Since  $z$  is the usual symbol for the vertical coordinate, as in  $dp = -\rho_0 g dz$ , inconsistency cannot be avoided.

#### A.2 SYMBOLS USED IN THE PRESENT REPORT

a	horizontal semi axis of cloud
b	vertical semi axis of cloud
$c_D$	drag coefficient
c	specific heat of liquid or solid water; speed of sound
$c_p$	specific heat of gas at constant pressure
$c_v$	specific heat of gas at constant volume
$E_k$	turbulent kinetic energy per unit mass
$e_s$	saturation vapor pressure of water
f	fraction of explosion energy, $W$ , contained in fireball at start of rise
g	acceleration of gravity
	relative humidity

$k_2$	empirical constant (in eddy viscosity)
$L$	latent heat of evaporation
$\ell$	characteristic length
$M$	molecular weight; Mach number
$m$	mass of cloud
$p$	pressure
$q(x)$	$\frac{1+x/\epsilon}{1+x}$
$R^*$	universal gas constant *
$R_a$	gas constant of air = $\frac{R}{M_a}$
$R$	weighted mean value of gas constant = $R_a \frac{1+x/\epsilon}{1+x}$
$r$	radius of cloud
$S$	surface area of cloud
$T$	temperature
$T^*$	$Tq(x)$ , i.e. virtual temperature
$t$	time
$u$	vertical velocity
$V$	volume of cloud
$v$	characteristic velocity, $v = \sqrt{u^2 + 2 E_k}$ or $\max( u , \sqrt{2E_k})$
$W$	total explosion energy (kilotons)
$w$	liquid and solid water mass per unit dry air mass
$x$	mixing ratio (water vapor mass per unit dry air mass)
$z$	vertical coordinate
$\alpha$	lapse rate of atmosphere = rate of decrease of temperature with height
$\beta$	ratio of gas density to total density of cloud = $\frac{1+x}{1+x+w}$
$\gamma$	ratio of specific heats $c_p/c_v$
$\epsilon$	ratio of molecular weights of water and air = 18/29
$\lambda$	empirical constant (in entrainment rate)
$\rho$	density

$\phi$  fraction of fireball energy used to heat air

SUBSCRIPTS

- a air (dry air)
- e ambient (environment) conditions
- h horizontal (radius of cloud)
- o initial condition
- w water or water vapor
- wv water vapor
- w $\ell$  liquid and solid water (i.e. water and ice)

## APPENDIX B

### CLOUD HEIGHT, CLOUD VELOCITY, AND ENTRAINMENT: POSSIBLE APPROACHES

A set of cloud-rise equations must contain a momentum equation if cloud properties are to be calculated as functions of time. Without a momentum equation, some of these properties, such as size and temperature can be determined as functions of height only, and dynamical properties such as turbulence characteristics cannot be given at all. By a static or quasi-static description of cloud rise we mean one lacking a momentum equation.

#### B.1 STATIC AND QUASI-STATIC METHODS

Static and quasi-static methods give a maximum cloud height--i.e. tell at what altitude, but not when, to stop cloud rise.

##### B.1.1 Static Equilibrium

In the static-equilibrium approach, cloud rise stops when cloud temperature equals ambient temperature. Actually the cloud may have enough momentum to rise far beyond this height, meanwhile entraining more air.

##### B.1.2 Quasi-Static Approach or Energy Balance

While the cloud is hotter or lighter than ambient, it loses potential energy and gains kinetic energy as it rises; when it is cooler or heavier the opposite energy change takes place (for a saturated cloud, latent-heat energy release is also considered).

Using this approach, Ref. 24 calculates temperature in a non-entraining "giant thunderstorm" cloud (observed top, 70,000 ft) by balancing the two areas between the ambient and the cloud (wet adiabatic)

temperature vs height curves, namely the areas above and below the intersection of the curves. The cloud picks up kinetic energy while it is warmer, and loses it (above the intersection) while it is cooler. This gives maximum height. For an (entraining) nuclear cloud, the areas would have to be weighted for variation of cloud mass with height.

Weighting the buoyancy vs height function,  $\Delta\rho(z)$ , according to cloud mass  $m$ , the cloud stops at the height  $z_1$  for which

$$\int_0^{z_1} m(z) \Delta\rho(z) dz = 0 \quad (B.1)$$

Here  $\Delta\rho = (\rho_e - \rho)/\rho_e$ , where  $\rho_e$  and  $\rho$  are, respectively ambient and cloud densities. Now the cloud is denser than the environment. To find an equilibrium height, one brings the cloud down again to  $z_2$ , for which

$$\int_{z_1}^{z_2} m \Delta\rho dz = 0 \quad (B.2)$$

and then repeats the integration upward to some  $z_3$ , etc.

If there were no entrainment (i.e.,  $m$  constant), this method would have the cloud oscillate indefinitely. Actually,  $m$  increases with time so that the amplitude of oscillation decreases. That is, entrainment exercises a braking effect. But the braking may be insufficient. Calculation with only this form of drag give excessive velocities and oscillations. (Refs. 17, 25, and calculations made with the set of equations given in this report.)

The additional drag force required can be expressed by a virtual mass, a drag coefficient, or as a coefficient of momentum exchange or momentum loss by turbulence. (Sec. 2.6.2)

The static and quasi-static approaches give minimum and maximum values of maximum cloud height, respectively. Both are energy methods and independent of time. The first disregards momentum completely. The second is an energy-integral method, and disregards any dissipative drag or friction, or either upward or downward motion.

## B.2 MOMENTUM EQUATIONS

When a momentum equation is used, if the cloud is "heavier than air" its rate of rise has decreased to zero, the cloud sinks until it again runs out of momentum (as it will in a stable atmosphere even without entrainment). Thus, the cloud performs damped oscillations. With sufficient damping, no oscillations occur because zero rate of rise is approached asymptotically.

If drag other than that due to entrainment is neglected, this approach gives the same buoyancy-height history as the quasi-static balance (App. B.1.2), but also the time history which quasi-static approaches cannot provide.

In a momentum equation, rate of change of cloud momentum (which allows for mass change) is balanced against buoyancy force, and perhaps some decelerating force other than mass change. Without the additional drag, the calculated rate of cloud rise may be excessively high.

Possibilities for the additional retardation term are:

1. Virtual mass (potential flow theory)
2. Drag coefficient
3. Momentum-exchange or loss coefficient.

### B.2.1 Potential Flow

The cloud is moving in a fluid that is neither homogeneous nor infinite, and the cloud itself is fluid, not a rigid body, and there is a dissipative loss of momentum to turbulence (Sec. 2.6.2). This suggests that use of potential flow theory for cloud rise must be modified. Nevertheless the potential flow approach may be useful.

In this problem there is not only accelerated flow, but also change in virtual mass,<sup>26</sup>  $m'$ , due to change both in "sphere" (cloud) volume and in the ratio  $T/T_e = \rho_e/\rho$ , where  $T$  and  $T_e$  are, respectively, cloud and ambient temperatures.

If these two changes are considered then

$$\frac{d}{dt} \left[ (m + m') u \right] = V (\rho_e - \rho) g \quad (B.3)$$

where  $u = \frac{dz}{dt}$ .

Since, for flow around a sphere, virtual volume is one half the displaced volume, i.e.,  $m' = (m/2) (\rho_e/\rho)$ , then

$$\begin{aligned} \frac{du}{dt} &= 2 \left( \frac{T - T_e}{T + 2T_e} \right) g - 2u \left( \frac{T_e}{T + 2T_e} \right) \frac{1}{m} \frac{dm}{dt} \\ &\quad - u \left( \frac{T_e}{T + 2T_e} \right) \frac{1}{m} \left( \frac{T}{T_e} \frac{dm}{dt} + m \frac{d(T/T_e)}{dt} \right) \end{aligned} \quad (B.4)$$

The last of the three terms on the right, which corresponds to the change in virtual mass, was omitted in Eq. (31) of Ref. 25.

If, as discussed at the end of Sec. 2.6.2, we allow only for an initial constant virtual mass,  $m' = m_0' = m_0 T_0 / 2T_{e0}$ , then Eq. (B.3) becomes

$$\frac{du}{dt} = \frac{m}{m + m_0'} \left[ \left( \frac{T}{T_e} - 1 \right) g - \frac{u}{m} \frac{dm}{dt} \right] \quad (B.5)$$

The effect of the factor  $m/(m + m_0')$  in the momentum equation, Eq. (3.1), of our set of cloud rise equations (Sec. 3.1), is to avoid unrealistically high initial acceleration. In Eq. (3.1) the presence of water vapor, as well as dry air, in the cloud and in the atmosphere is taken into account, so that the temperatures  $T$  and  $T_e$  in the momentum equation are replaced by the corresponding virtual temperatures  $T^*$  and  $T_e^*$ .

The potential flow formula for rise of a bubble with a spherical leading face<sup>4,27</sup> agrees surprisingly well with observations of early cloud rise, for yields up to 14.5 MT. At this and higher yields there may be local transonic flow around equator of cloud and additional pressure drag. (See Sec. 2.6.4) The equation for steady-state rate of rise of an empty bubble, of radius  $r$ , is

$$u = \frac{2}{3} \sqrt{gr} \quad (B.6)$$

The generalization of this to non-empty bubbles is

$$u = \frac{2}{3} \sqrt{\frac{gr(\rho_e - \rho)}{\rho_e}} \quad (B.7)$$

If we accept this rate of rise as equal to the observed rate of rise, then from observed cloud rise, one could estimate the density ratio  $\rho/\rho_e$ .

For an ellipsoidal cloud, instead of horizontal radius  $r$ , the radius of curvature,  $r^2/b$ , may be used, where  $b$  is the vertical radius of the cloud.

### B.2.2 Drag Coefficients

Writing a momentum equation for the cloud, with a drag term, such as given by Eq. (2.12) in Sec 2.6.2,

$$\frac{d}{dt} (\mu u) = V(\rho_e - \rho) g - C_D \left( \frac{1}{2} \rho_e u^2 \right) \pi r^2 \quad (B.8)$$

where  $V$  is the cloud volume, so that for a sphere

$$\frac{d}{dt} (r^3 \rho u) = r^3 (\rho_e - \rho) g - \frac{3}{8} C_D r^2 \rho_e u^2 \quad (B.9)$$

If changes in  $r$  and  $u$  are neglected, then

$$u = \sqrt{\frac{8 gr(\rho_e - \rho)}{3 C_D \rho_e}} \quad (B.10)$$

Note the similarity between Eq. (3.10) and the potential-flow equation, Eq. (B.7). Allowing for acceleration and mass change, since  $u = \frac{dz}{dt}$ ,

$$r(\rho_e - \rho) g - \frac{3}{8} C_D \rho_e u^2 = u^2(3\rho \frac{dr}{dz} + r \frac{dp}{dz}) + r\rho u \frac{du}{dz} \quad (\text{B.11})$$

The two "mass entrainment" terms in parentheses on the right side of Eq. (B.11) act as additional drag terms.

At least for yields of 100-KT and more, cloud diameter approaches or exceeds cloud height, so that in the later stages of cloud rise  $\frac{dr}{dz} \geq 1/2$ . Since also then  $\rho \approx \rho_e$  and  $C_D$  is about 0.5,

$$3\rho \frac{dr}{dz} \gg \frac{3}{8} C_D \rho_e$$

That is, the drag-coefficient term makes a minor contribution to the total drag. This is confirmed by Ref. 5. But for high yields, early in cloud rise, where we can assume initial cloud density to be about 10% that of ambient air, calculated drag coefficients (to fit observed rate of rise) may be greater than typical high-Reynolds-number values for spheres of 0.5 or less. As with the breakdown of potential flow formulas, one suspects the influence of compressibility. (Sec. 2.6.3)

In any case, the use of the drag concept is questionable. The cloud is a fluid body, and its boundary layer is largely entrained, so that although cloud momentum is lost total kinetic energy is not. This suggests the use of a momentum-exchange or loss function to describe the kinetic energy but not the exact flow pattern of the cloud. Certain entrainment and eddy-viscosity ideas lead to the same mathematical form of the momentum equation as above, but with more physical-intuitive conviction (Sec. 2.6).

### B.2.3 Momentum Exchange and Momentum Loss

B.2.3.1 External Exchange. One possible momentum-exchange rule is that, per unit time, a certain fraction of cloud mass is exchanged for an

equal mass (or possibly an equal volume) of ambient air. Since the ambient air is stationary there is a net loss of cloud momentum per mass (i.e., a dilution of momentum) even though mass is not lost. Physically: a parcel of ambient air is entrained by the cloud. Part of the parcel stays in the cloud to increase cloud mass. The rest of the parcel is "mixed out," i.e. it returns to the ambient air, taking with it some of the cloud momentum.

If the "mixed-out" air has been in the cloud long enough to acquire momentum, should it not acquire heat too? The answer is: not if heat transfer requires more thorough mixing. Picture the ambient parcel as entrained and broken into two parts, and one part as ejected. The ejected part has gained momentum, but its interior is undisturbed, so no heat transfer has occurred.

What happens to the lost momentum represented by an exchange term in the momentum equation? The implication is that it is eventually dissipated in the atmosphere without perturbing cloud rise.

The treatment of exchange factor as a constant rather than velocity-dependent is appropriate to environmental turbulence.<sup>28</sup> For the rising atomic cloud, any environmental turbulence is negligible compared with cloud-induced turbulence as an influence on exchange coefficients. In any event, the exchange factor should be less for large than for small clouds unless increased circulation compensates for the smaller area-to-volume ratio.

A semi-theoretical justification for Anderson's<sup>8</sup> cloud rise equation is as follows: in Sutton's<sup>2</sup> diffusion type equation we take the exponent 1.88 of altitude,  $z$ , as  $\approx 2$ , then we get

$$z \propto t^{1/1.88} w^{(1/2)(1/1.88)}, \text{ or } z \propto t^{1/2} w^{1/4}$$

as in Ref. 8. Taking 1.88  $\approx 2$  in Sutton's equation means assuming zero atmospheric turbulence, which is reasonable relative to cloud turbulence.

B.2.3.2 Internal Exchange. Suppose momentum exchange takes place "inside" the cloud rather than on its surface, or that the surface exchange is with air all of which is promptly entrained.

For surface exchange, the sequence would be: (1) internal motion causing momentum exchange near the cloud surface, (2) circulation of fluid "lumps" from front and sides to rear surface (base) of cloud, (3) entrainment of more fluid, (4) loss of momentum and of kinetic energy of rise, and (5) reentry of the randomly moving lumps into the cloud.

Putting these ideas together suggests that the exchanged momentum and the corresponding kinetic energy of directed motion are not simply dumped into the atmosphere, but go into cloud turbulence. The narrowness of the wake in nuclear-cloud photographs also suggests this internal disposal process.

The momentum is not all transferred directly to turbulent kinetic energy. Some of it must go to setting up the observed vortex circulation. But the vortex eventually disappears with degradation of its motion to turbulence, i.e., to smaller, fluctuating vortices or finally to thermal energy; so in calculating the effect of exchanged or "lost" kinetic energy of rise, or accumulated energy of turbulence, on the horizontal cloud diffusion after end of rise, we may ignore the vortex formation and breakdown as intermediate steps in the energy-conversion process.

The set of cloud-rise equations in Sec. 3 uses an internal exchange mechanism. Since this mechanism is closely related to the entrainment mechanism, the two processes are discussed together (Sec. 2.6).

### B.3 ENTRAINMENT

One way to idealize entrainment is to consider it as a snowball mechanism, i.e. rolling contact entrainment. Then the question arises: how thick should the layer of air that the cloud rolls up be? Presumably it should be proportional to the ratio of cloud density to

ambient density, as discussed in Sec. 2.6.1. If, aside from that, it is constant, we get

$$\frac{1}{V} \frac{dV}{dz} \sim \frac{1}{r^2}$$

where  $V$  = cloud volume, and  $r$  = cloud radius. If thickness is inversely proportional to the two-way snowball curvature, we get

$\frac{1}{V} \frac{dV}{dz} = \text{constant}$  (except for the density ratio); i.e.,  
Machta's model.<sup>1</sup>

Another entrainment rule is that rate of cloud mass increase is proportional to rate of rise times cloud surface area.<sup>4, 18</sup> This rule, with modifications is used in our cloud rise equations, in Sec. 3. Since it is closely related to our model of momentum change, the two subjects are discussed together in Sec. 2.6.

## APPENDIX C

### PHYSICAL CONSTANTS AND ENVIRONMENTAL AND INITIAL CONDITIONS USED IN NUMERICAL SOLUTION OF THE EQUATIONS

All quantities are in the mks system, except explosion energy  $W$ , which is in kilotons (KT): 1 KT =  $4.18 \times 10^{12}$  joules.

#### C.1 PHYSICAL CONSTANTS

$$g = 9.80$$

$$L = 2.50 \times 10^6 \text{ for vapor to liquid transition}$$

$$L = 2.83 \times 10^6 \text{ for vapor to solid transition}$$

$$R_a = 287$$

$$\epsilon = 18/29$$

#### C.2 ENVIRONMENTAL CONDITIONS

$T_e = T_{e,i-1} - \alpha_i (z - z_{i-1})$  for the  $i$ th layer of a 4-layer atmosphere defined recursively by sea-level temperature  $T_{eo}$ , the dividing heights 0,  $z_1$ ,  $z_2$ ,  $z_3$ , and the lapse rates  $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ .

$$p = p_{i-1} \left( \frac{T_e}{T_{e,i-1}} \right)^{g/R_a \alpha_i} \quad \text{for the } i\text{th of the 4 layers.}$$

Thus  $p$  is defined recursively.  $p_0$  is the only independent pressure parameter, besides those specifying  $T_e$ .

$T_{eo}$  and  $p_0$  are the values at sea level,  $z = 0$ , and not at initial height  $z_0$ , unless  $z_0 = 0$ .

Note that isothermal layers,  $\alpha_1 = 0$ , can not be used, but can be approximated by small absolute  $\alpha$ , say  $\alpha = .0001$ .

$x_e = h_e \in e_s/p$ , where  $h_e$ , relative humidity, is given by a quadratic polynomial in  $z$  for

$$z \leq 10^4 \text{ and is taken as 0 for } z > 10^4$$

$e_s$ , saturation vapor pressure, is given by the integrated Clausius - Clapeyron equation:

$$e_s = 611 \cdot (T/273)^{-5.13} e^{25.0(T-273)/T}$$

### C.3 INITIAL CONDITIONS

Values are assigned to  $W$ ,  $f$ ,  $\phi$ ,  $z_o$ . These values, together with the environmental conditions, determine the initial values of  $m$ ,  $x$ ,  $V$ ,  $r$ .

$$m_o = m_{ao} + m_{wo}, \text{ where}$$

$$m_{ao} = \frac{\phi f W (4.18 \times 10^{12})}{\int_{T_{eo}}^{T_o} c_{pa}(T) dT}$$

$$m_{wo} = \frac{\phi f W (4.18 \times 10^{12})}{\int_{T_{eo}}^{T_o} c_{pw}(T) dT + L}$$

$c_{pa}$  and  $c_{pw}$  are quadratic polynomials in  $T$  whose coefficients are parameters to be specified.

Strictly,  $T_e(z_o)$  should be used in  $m_{ao}$  and  $m_{wo}$ , instead of  $T_{eo}$ , but the difference (when  $z_o \neq o$ ) is unimportant because  $(T_o - T_{eo})$  is large compared to  $(T_e(z_o) - T_{eo})$ .

The initial values of  $x$ ,  $V$ , and  $r$  are given by

$$x_0 = m_{wo}/m_{ao}$$

$$V_0 = m R_a T_0^* / p(z_0)$$

$$r_0 = (3V_0/4\pi)^{1/3}$$

## APPENDIX D

### GLOSSARY OF COMPUTER PRINTOUT SYMBOLS

Symbol in printout	Symbol in text	Comment
<u>Output Symbols</u>		
ST	t	
U	u	
X	x	
T	T	
R	r	After SWITCH TO ELLIPSE R refers to horizontal radius, vertical radius remaining fixed
Z	z	
EK	$E_k$	
V	v	
WT	w	
TE	$T_e$	
M	m	
ES	$e_s$	Saturation water vapor pressure at T. Values of ES printed for $T > 373$ have no physical significance

Symbol in printout	Symbol in text	Comment
P	p	
PW	$\frac{px/\epsilon}{1 + x/\epsilon}$	partial pressure of water vapor in cloud
ED	$k_2 \frac{u^2  v }{\ell} \frac{T^*}{T_{e^*}}$	rate of loss of kinetic energy of rise due to eddy viscosity, per unit mass.

Parameter Symbols

DST		Runge-Kutta step size between $t = 0$ and $t = 1$ .
K2	$k_2$	
LAMBDA	$\lambda$	
C1, C2, C3		Coefficients of polynomial giving relative humidity, $h_e$ , between $z = 0$ , $z = 10000$ : $h_e = C1 - (C2)z - (C3)z^2$
W	w	
F	f	
PHI	$\phi$	
TEO	$T_{eo}$	Sea-level temperature
CHANGE		Value of ST after which Runge-Kutta step size changes to DST2
DST2		Runge-Kutta step size when $ST > CHANGE$

Symbol in printout	Symbol in text	Comment
B0, B1, B2		Coefficients of quadratic polynomial in T for $c_{pa}$ .
DST1		Runge-Kutta step size for $1 < ST < \text{CHANGE}$
K		Option in program. K = 1, 3 select the "alternate" equations, K = 1, 2 select the characteristic length $\ell = r$ . K = 3, 4 select $\ell = 100$ . This last value of $\ell$ is in a dummy equation in which some other value of $\ell$ , such as a Mach-number correction, can be inserted.
A1, A2, A3, A4 $\alpha_1, \alpha_2, \alpha_3, \alpha_4$		Lapse rates in the 4 layers of the atmosphere
Z1, Z2, Z3 $z_1, z_2, z_3$		Dividing altitudes of the 4-layer atmosphere
P0	$p_0$	Sea level pressure
ZT		Altitude such that if $Z > ZT$ (SWITCH TO ELLIPSE) cloud vertical radius remains constant (nominal height of tropopause)
D0, D1, D2		Coefficients of quadratic polynomial in T for $c_{pw}$ .
TF		Freezing point used to select value of L (after SWITCH TO WET)
C3		See C1, C2

Symbol in printout	Symbol in Text	Comment
PRINT		End time of computation
RK3		If RK3 = 1 the momentum equation contains the initial virtual mass factor. If RK3 = 0 this factor is omitted.

## APPENDIX E

### COMPUTER PROGRAM

The following program (App. E-2) has been run on an IBM 7094. An IBM 704 version is also available. The program was written by David Hutchinson of C-E-I-R, Inc.\*

#### E.1 COMMENTS ON THE PROGRAM (By David Hutchinson)

##### General Remarks.

The integrator used is "Illinois Library Routine D2-DRKL-45-SR" "Double Precision Floating Point Runge Kutta." Double precision was not needed; however, the routine was handy and had been checked out by the programmer at the University of Illinois in August 1963. The extra time required to do the 7094 double-precision operations is negligible compared to the time of the DERIV subroutine which is single-precision FORTRAN coded. The equations are evaluated in double precision (in the 7094 version).

In the 704 version of the integrator (which will run on a 709/90/94), the double-precision 7094 instructions have been replaced by the corresponding single-precision instructions. Now the equations are evaluated in single-precision. This will cause a slight deterioration in accuracy on long runs, but not enough to affect the number of digits printed in this problem.

Because the integrator is double-precision and was not designed to be used with FORTRAN, the FORTRAN program might look curious to some.

---

\*Now at the University of Illinois, Urbana.

All the variables and their derivatives occupy two core locations,  $2n$  and  $2n + 1$ . The more significant part is in  $2n$  and the less significant part, which is zero usually, is in  $2n + 1$ . This is so because the 7094 double-precision instructions require the more significant part in an even core location. Because FORTRAN II stores its arrays in decending core locations, the derivative  $\frac{du}{dt}$ , for example, has its more significant part put in  $DU(2)$  by the DERIV subroutine. The less significant part is zero which is put into  $DU(1)$  once and for all in the MAIN routine.

Punching the data cards.

All the data are punched in the column F fields except K which is punched in a 1 column I field (column 71 on card 3). At the Oakland installation of C.E.I.R. (where this program was debugged) the version II FORTRAN monitor is used and zeros are punched as 0.0. If a blank field is read (it should be read as minus zero for this program to run correctly) then the field is ignored and the value of the datum which was last read in is used. Hence, data which do not change from one run to the next need not be punched again, except for card 1 where all 7 fields must be punched each time. The five data cards are punched as follows:

Cols	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80
Card 1	U	RK3	T	blank	Z	EK	DST	
Card 2	K2	$\lambda$	C1	C2	W	F	$\emptyset$	
Card 3	TEO	CHANGE	DST2	BO	B1	B2	DST1	K (must be in col 71)
Card 4	A1	A2	A3	A4	Z1	Z2	Z3	PO
Card 5	ZT	DO	D1	D2	TF	C3	PRINT	

## HUTCHINSON-HUEBUSCH NAVAL RADIOLOGICAL LAB.

## DIFFUSION DECK

#	FAP	
	COUNT 90	
DLD	OPD 044371100000	
DST	OPD 460371100000	
DFAD	OPD 030171100000	
DFMP	OPD 026171100000	
* UOI DOUBLE PRECISION FL. PT. RUNGE-KUTTA-GILL		
	ENTRY DRK1	
	NOP	
* REPLACE THIS CARD BY EVEN PSEUDOP WHEN IT BECOMES OPERATIVE		002
DRK1	SXA SR,1	
	SXA SR+1,2	
	NZ1# 6,4	
	TRA NEW	
	AXT 8,1	
	SXA SX4,4	
EVAL	CALL DERIV	
	AXT **,2	
NLOOP	DLD **,2	
	DFMP **	
	DST **,2	13
	DLD B+8,1	
	DFMP **,2	15
	DFAD **,2	016
	DFMP A+8,1	
	DST TS	
	DFAD **,2	19
	DST **,2	20
	DLD **,2	021
	DFMP C+8,1	
	DFAD TS	
	DFAD TS	
	DFAD TS	
	DFAD **,2	26
	DST **,2	27
	TIX NLOOP,2,2	028
	TIX EVAL,1,2	029
SR	AXT **,1	30
	AXT **,2	31
SX4	AXT **,4	32
	TRA 7,4	33
NEW	CLA# 5,4	34
	STO# 6,4	35
	STA NLOOP-1	036
	STA ZQ-1	37
	ADD 4,4	38
	STA NLOOP+4	039
	STA NLOOP+15	040
	STA NLOOP+16	041
	STA ZQ	42
	CLA 1,4	43
	ADC# 5,4	44
	STA NLOOP+8	045
	STA NLOOP+9	046
	CLA 2,4	47
	ADD# 5,4	48
	STA NLOOP	049
	STA NLOOP+2	050
	STA NLOOP+5	051
	STA NLOOP+10	052

```

      CLA      3,4          53
      STA      NLOOP+1       054
      AXT      **,1           55
      STZ      **,1           56
      TIX      ZQ,1,1         57
      TRA      EVAL-2        #58
      A       OCT 200400000000,145000000000 59
      OCT 177453730314,144600610316 060
      OCT 201665011714,146637635715 061
      OCT 176525252525,143252525253 062
      B       OCT 602400000000,547000000000 63
      OCT 601400000000,546000000000 64
      OCT 601400000000,546000000000 65
      OCT 602400000000,547000000000 66
      C       OCT 600400000000,545000000000 67
      OCT 577453730314,544600610316 068
      OCT 601665011714,546637635715 069
      OCT 600400000000,545000000000 70
      TS      BSS 2           71
      END
      *      FORTRAN
      C      THIS IS THE MAIN ROUTINE

      DIMENSION PAR(37),A(74),Q(16),DU(2),DX(2),DT(2),DV(2),DZ(2),DEK(2)
      2,SMALLT(2),DRM(2),DUMMY(4),DWT(2)

      COMMON DUMMY,A,DWT,DRM,DU,DX,DT,DV,DZ,DEK,SMALLT,Q,TE,P,P3,P1,P2,N
      2,TE1,TE2,TE3,ED,NZT,RZT,RK3,QI
      EQUIVALENCE (DUMMY(2),WT),(DUMMY(4),RM)
      1      , (A(2),U),(A(4),X),(A(6),T),(A(8),V),(A(10),Z),(A(12),EK)
      2,(Y),(A(14),DST),(A(16),RK2),(A(18),RL),(A(20),C1),(A(22),C2),(A(24)
      3),W),(A(26),F),(A(28),PHI),
      4(A(30),TE0),(A(32),CHANGE),(A(34),DST2),(A(36),B0),(A(38),B1),
      5(A(40),B2),(A(42),DST1),(A(44),K)
      6,(A(46),A1),(A(48),A2),(A(50),A3),(A(52),A4),(A(54),Z1),(A(56),Z2)
      7,(A(58),Z3),(A(60),P0),(A(62),ZT),(A(64),D0),(A(66),D1),(A(68),D2)
      8,(A(70),TF),(A(72),C3),(A(74),PRINT)

      C      INPUT THE INITIAL CONDITIONS FOLLOWED BY DST AND PARAMETERS
      1      READ INPUT TAPE 5,19,PAR
      19     FORMAT(7F10.0/7F10.0/7F10.0,I1/8F10.0/8F10.0)
      C      IF T=PAR(3) IS NEGATIVE IT IS A SIGNAL TO STOP
      18     IF(PAR(3))18,4,4
      CALL DUMP
      4      DO 3 I=1,37
      C      IF FIELD IS BLANK USE LAST PARAMETER VALUE INPUT
      1     IF(PAR(I))2,33,2
      B 33   D=PAR(I)+201400000000
      1     IF(D) 3,2,2
      2     A(2*I)=PAR(I)
      3     A(2*I-1)=0.
      RK3=X
      C      PRINT THE PARAMETERS
      WRITE OUTPUT TAPE6,17,(A(I),I=14,74,2),RK3

```

```

17   FORMAT(5H1DST=F8.5,5H K2=F10.7,9H LAMBD=10.6,5H C1=F11.7,5H
      2C2=F11.7,4H W=E12.6,4H F=F12.8,6H PHI=F12.8/
      3 7HO TEO=F10.6,9H CHANGE=F7.3,7H DST2=F10.4,5H B0=F11.2 ,
      4 5H B1=F11.4,5H B2=E12.4,7H DST1=F7.1,4H K=11/6HO A1=F8.5,
      55H A2=F8.5,5H A3=F8.5,5H A4=F8.5,5H Z1=F8.0,5H Z2=F8.0,5H Z3
      6=F8.0,5H P0=F9.0/6HO ZT=F8.0,5H D0=F8.1,5H D1=F8.5,5H D2=F10,
      78,5H TF=F8.1,5H C3=F8.6,8H PRINT=F6.1,6H RK3=F4.1/1HO)
B   BN=000000000020
     FL=0.
     SMALLT(1)=0.
     SMALLT(2)=0.
     DWT=0.
     DRM=0.
     DU=0.
     DX=0.
     DT=0.
     DV=0.
     DZ=0.
     DEK=0.
     TO=T
     DWT(2)=0.
     TE=TE0
     TE1=TE0-A1*Z1
     TE2=TE1-A2*(Z2-Z1)
     TE3=TE2-A3*(Z3-Z2)
     XX=9.8/287.
     P1=P0*(TE1/TE0)**(XX/A1)
     P2=P1*(TE2/TE1)**(XX/A2)
     P3=P2*(TE3/TE2)**(XX/A3)
     RMA0=PHI*F*W*4.18E12/(B0*(TO-TE0)+B1/2.* (TO*TO-TE0*TE0)
2      +B2/3. *(TO*TO*TO - TE0*TE0*TE0))
     RMWO=(1.-PHI)*F*W*4.18E12/( DO *(TO-TE0)+ D1/2.* (TO*TO-TE0*TE0) +
2      D2/3. *(TO*TO*TO - TE0*TE0*TE0)+2500000.)
C   SET INITIAL COND. FOR X
     X = RMWO/RMA0
C   SET INITIAL FOR V
     P=P0*((TE0-A1*Z )/TE0)**(XX/A1)
     V=(RMA0+RMWO)*287.*TO*(1.+29.*X/18.)/( P*(1.+X))
C   SET INITIAL COND. FOR R
     R=(3.*V/12.5663706)**.333333333
     RM=RMA0+RMWO
     Q1=.5*RM*T*(18.+29.*X)*(1.+XE)/(TE*(18.+29.*XE)*(1.+X))
     N=1
     WT=0.
     RZT=-1.0
     WRITE OUTPUT TAPE 6, 99
99   FORMAT(4H ST, 7X,4HU ,7H X ,9H T ,10H R ,
2 9H Z ,8H EK , 5HV ,8H WT ,9H TE ,12H
3 M ,10H ES ,8H P ,10H PW ,9H ED /1H )
C   PRINT INITIAL CONDITIONS
     GO TO 35
B   PU=U
     PX=X
     PT=T
     PZ=Z
     PEK=EK
     PV=V
     PRM=RM

```

```

C           TAKE A RUNGE-KUTTA STEP
CALL DRK1(Y,DEK(2),DST,Q(16),UN,FL)

C           SMALLT(2)=SMALLT(2)+DST
C           TAKE SMALL STEPS INITIALLY
87 DST=DST1
88 IF(RZT)888,89,89
89 R=SQRTF(3.*V/(RZT*12.5663706))
GO TO 35
888 R=(3.*V/12.5663706)**.333333333

35 PW=P*X*29./(18.+29.*X)

ES=611.* (T/273.)**(-5.13)*EXP((25.*(T-273.))/T)

11 WRITE OUTPUT TAPE 6,16,SMALLT(2),U,X,T,R,Z,EK, V ,WT ,TE, RM,
2ES,P,PW,ED
16 FORMAT(F6.1,F 8.1,F9.4,F8.1,F 8.0,F8.0,F7.0,E11.3,F8.4,F7.1,E11.3
2 ,F10.0,F9 .0,F8.0 ,F12.3)

C           N=1, DRY MODE   N=2, WET MODE   N=3, SMALL STEP DRY MODE
150 IF (N-2) 150, 154, 1531
IF (ES-PW) 152,152,151

152 SAVE=DST
DST=0.5
C           RESTORE VARIABLE VALUES AT START OF LAST STEP
SMALLT(2)=SMALLT(2)-SAVE
U=PU
X=PX
T=PT
Z=PZ
EK=PEK
V=PV
RM=PRM
N=3

C           NOW TAKE SMALL STEPS UNTIL ES LESS THAN PW
GO TO 8

1531 IF (ES-PW)41,41,8
41 DST=SAVE
N=2
WRITE OUTPUT TAPE 6,77
77 FORMAT(14HOSWITCH TO WET)
GO TO 151
154 IF(WT+.00000001) 153,153,151
153 N=1
WT=0.
DWT=0.0
DWT(2)=0.0
WRITE OUTPUT TAPE 6,66
66 FORMAT (14HOSWITCH TO DRY)
C           IF RZT IS POSITIVE WE ARE IN ELLIPSOIDAL MODE
151 IF(RZT)50,1511,1511
C           SWITCH TO ELLIPSOIDAL IF Z LARGER THAN ZT
50 IF(Z-ZT)1511,51,51
51 RZT=R
WRITE OUTPUT TAPE 6,52
52 FORMAT(24H SWITCH TO ELLIPSE, R=RH)

```

```

1511 IF(SMALLT(2)-CHANGE) 14, 15,14
15 DST=DST2
14 IF(ABSF(T)-10.) 1,20,20
20 IF(R-1.) 1,21,21
21 IF (SMALLT(2)-10.)22,210,210
210 IF(ABSF(DU(2))-1) 211,22,22
211 IF(ABSF(U)-1) 1,22,22
22 IF (SMALLT(1)-PRINT)13, 13, 1
13 IF(Z-10000.*W**.25)8,8,1
END
*
FORTRAN
SUBROUTINE DERIV
C THIS PROGRAM COMPUTES THE DERIVATIVES FOR DRK1
C THIS PROGRAM ENTERED 4 TIMES FOR EACH R-K STEP
DIMENSION PAR(37),A(74),Q(16),DU(2),DX(2),DT(2),DV(2),DZ(2),DEK(2)
2,SMALLT(2),DRM(2),DUMMY(4),DWT(2)

COMMON DUMMY,A,DWT,DRM,DU,DX,DT,DV,DZ,DEK,SMALLT,Q,TE,P,P3,P1,P2,N
2,TE1,TE2,TE3,ED,NZT,RKT,RK3,QI

EQUIVALENCE (DUMMY(2),WT),(DUMMY(4),RM)
1           +(A(2),U),(A(4),X),(A(6),T),(A(8),V),(A(10),Z),(A(12),EK
2,Y),(A(14),DST),(A(16),RK2),(A(18),RL),(A(20),C1),(A(22),C2),(A(24
3),W),(A(26),F),(A(28),PHI),
4,(A(30),TEU),(A(32),CHANGE),(A(34),DST2),(A(36),B0),(A(38),B1),
5,(A(40),B2),(A(42),DST1),(A(44),K),
6,(A(46),A1),(A(48),A2),(A(50),A3),(A(52),A4),(A(54),Z1),(A(56),Z2)
7,(A(58),Z3),(A(60),PU),(A(62),ZT),(A(64),D0),(A(66),D1),(A(68),D2)
8,(A(70),TF),(A(72),C3),(A(74),PRINT)
DZ(2)=U
C COMPUTE TE AND P
XX=9.8/287.
IF(Z-Z1)80,80,81
80 TE=TE0-A1*Z
P=PU*(TE/TE0)**(XX/A1)
GO TO 89
81 IF(Z-Z2)82,82,83
82 TE=TE1-A2*(Z-Z1)
P=P1*(TE/TE1)**(XX/A2)
GO TO 89
83 IF(Z-Z3)84,84,85
84 TE=TE2-A3*(Z-Z2)
P=P2*(TE/TE2)**(XX/A3)
GO TO 89
85 TE=TE3-A4*(Z-Z3)
P=P3*(TE/TE3)**(XX/A4)
89 CONTINUE

62 IF(Z-10000.)36,36,37
37 XE=0.
GO TO 38
36 XE = 18.*(C1-C2*Z-C3*Z*Z)*
1           611.*(TE/273.)**(-5.13)*EXP((25.*(TE-273.))/TE
2)/(P*29.)
38 CPAI=B0*(T-TE)+B1/2.* (T*T-TE*TE)+B2/3.* (T*T*T-TE*TE*TE)

```

```

CPW=DU + D1*T + D2*T*T
CP=(B0+B1*T+B2*T*T + X*CPW)/(X+1.)
QXE=(1.+XE)/(1.+29.*XE/18.)
QX=(1.+29.*X/18.)/(1.+X)
QT=T/TE
IF(RZT)35,70,70
35 R~(3.*V/12.5663706)**.333333333
SV=3./R
GO TO 49
70 R=SQRTF(3.*V/(RZT*12.5663706))
ECC=SQRTF(R*R-RZT*RZT)/R + 1.0E-15
SV=3.1415926*(2.*R*R+RZT*RZT)*LOG((1.+ECC)/(1.-ECC))/ECC)/V
J=K
999 Q7= MAX1F(ABSF(U),SQRTF(2.*EK))
GO TO (50,50,51,51),J
DIFUSION
50 IF(RZT)53,54,54
53 R2=R
GO TO 52
54 R2=RZT
GO TO 52
51 R2=100.
52 GO TO (60,61,60,61),J
60 DRM(2)=SV*Q7*(1.+29.*X/18.)*T *RL      *RM*QXE/((1.+X+WT)*TE) DIFUSION
QQ=1.0
GO TO 621
61 DRM(2)= SV*RM*Q7*RL
QQ = QT*QX*QXE*(1.+X)/(1.+X+WT)
DIFUSION
621 DU(2)=(9.8*( QT*QX *(1.+X)/(1.+X+WT)*QXE - 1. ) - (QQ* Q7 *2.*RK2/R2 + DRM(2)/RM)*U )*
3 (RM+QI*(1.-RK3))/(RM+QI)
M=N
GO TO (100,101,100),M
100 DX(2)=-(1.+X)*(X-XE)*DRM(2)/(RM*(1.+XE))
DT(2)=-QX*QT*9.8*U/CP * QXE          - CPAI*DRM(2)/(CP*RM)
C                                     RK2 IS K2, RL IS LAMBDA, RM IS M
RK2 IS K2, RL IS LAMBDA, RM IS M

DV(2)=(9.8*QXE*
2   (1./(287.*QX)-1./CP)*QX*U/TE+(1.-CPAI/(T*CP))*DRM(2)/RM +
3  11.*DX(2)/(18.*QX*(1.+X)*(1.+X) ) )*V

GO TO 555
C THIS IS THE WET PART
101 Q1 = 1. + X*29./18.
IF(T-TF)102,103,103
102 CL=2.83E6
GO TO 104
103 CL=2.5E6
104 Q2 = CL*X/( 287.*T )
Q3 = 18.*C2/( T*29. )
Q4 = 1. + Q2
Q5 = 1.+ CL*Q3/CP
Q6 = CL*(X-XE)/CP + T-TE
DT(2)= (-QX*QT*9.8*Q4*U/CP *QXE -Q6*DRM(2)/RM)/Q5
DX(2) = Q1*(Q3*DT(2) + 9.8*X*U/(287.*TE)*QXE)
DV(2)=V*( 9.8*(1./(287.*QX)- 1./CP)*QX*Q4*U/(Q5*TE) *QXE
1 + (1.- Q6/(T*Q5))*DRM(2)/RM)
DWT(2)=-( 1.+X+WT)*(WT+X-XE)*DRM(2)/((1.+XE)*RM) - DX(2)
555 ED=RK2* U*U*Q7 *QQ/R2
DEK(2)= ED - EK*DRM(2)/RM
RETURN
END

```

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